Reliability evaluation of engineering systems

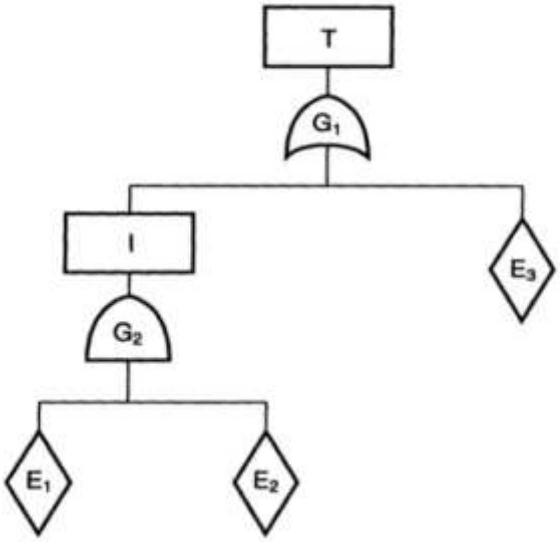

Fig. 5.17 Redrawn fault tree of Figure 5.15. T, top event – loos of electric power; G_1G_2, logic gates; I, intermediate event – loss of a.c. power; E_1, event 1 – loss of offsite power; E_2, event 2 – loss of onsite power; E_3, event 3 – loss of d.c. power

The probability of occurrence of the top event T can be readily evaluated using the rules of probability defined in Section 2.3, i.e.,

$$P(T) = P\,(E_1E_2 + E_3)$$

$$= [P(E_1)\,P(E_2)] + P(E_3) - \{P(E_1)\,P(E_2)\,P(E_3)$$

Where

$P(E_1) = 1 - 0.933 = 0.067$

$P(E_2) = 1 - 0.925 = 0.075$

$P(E_3) = 1 - 0.995 = 0.005$

Therefore

$P(T) = 0.01$

This represent the probability of ‘loss of electric power’ , previously used for the value of Q(EP) in Example 5.6.

This example is simple and straightforward since the number of basic events and hierarchical levels is very small and all events are independent. In practice this is not necessarily the case and related expressions can become very extensive and virtually impossible to deduce. However, the concepts contained in this example are general and apply in principle to all types of fault trees.

Network modelling and evaluation of complex systems

(b) Direct numerical approach

The main disadvantage of the Boolean algebra approach is the complexity of the expressions when large system and associated fault trees are being assessed. An alternative approach is to evaluate numerical values of probability during the reduction process of the fault tree rather than leaving this evaluation until after the top events has been expressed by a single statement. This numerical approach is a bottom-up method, whereas the previous one is a top-down.

The numerical approach starts at the lowest hierarchical level and combines event probabilities at this level using the appropriate logic gate by which they are connected. This combined probability gives the probability of the intermediate event at the next hierarchical process continues upwards until the top events is reached.

Example 5.10

Evaluate the probability of 'loss of electric power' as in Example 5.9 using the numerical approach:

$P(I) = P(E_1)\,P(E_2)$

$= (1 – 0.933)(1 – 0.925)$

$= 0.005025$

$P(T) = P(I) + P(E_3) – P(I)P(E_3)$

$= 0.005025 + (1 – 0.995) – 0.005025(1 – 0.995)$

$= 0.01$ (as before)

The concept used in this numerical example can be continued upwards through many hierarchical levels simply using the principles of logic gates and basic rules for combining probabilities. It is evident therefore that the advantage of this method is that it is a gradual reduction process which enables the probabilities of equivalent (intermediate) events to be evaluated sequentially and prevents extensive logical statements being created.

It has one particular disadvantage, however, which is not evident in the present example. This is when a basic event occurs more than once in the same fault tree. In such cases, the numerical approach may allow this basic event to be counted more than once, which clearly will lead to erroneous results. This problem is discussed further in the next section.

It is also worth noting that there are many commercially available computer programs and codes that solve fault trees. It is not appropriate for the authors to comment on the merits of these alternative programs.

Reliability evaluation of engineering system

5.8.4 Duplicated basic events

There are many occasions in real system where a component has an effect in the behavior of several other component or subsystems. A typical example could be the battery source or d.c. supply of a protection or control system in which several functions of the system are affected if the d.c. supply could appear several times in the same fault tree. Careful thought is needed if these duplicated basic events are not to cause significant erroneous result. The principles involved can best be illustrated using the following example.

Example 5.11

Consider the fault tree shown in figure 5.18. Evaluate the probability of occurrence of the top event T when:

(a) all basic events are independent of each other

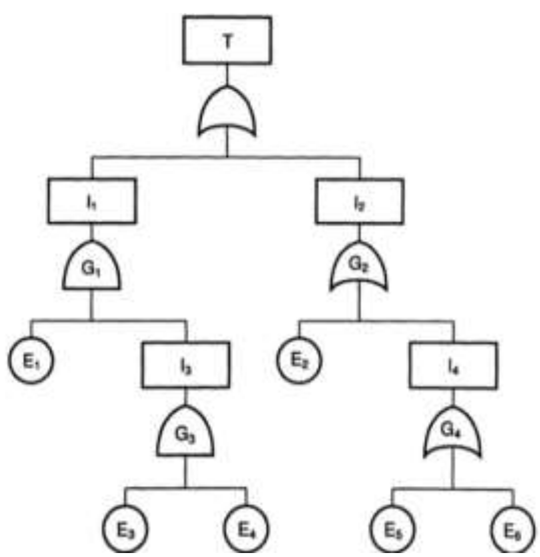

Fig. 5.18 fault tree for Example 5.11

(b) basic event E_3 is the same as basic event E_6, i.e., these events represent the same failure mode of the same component.

Let the basic event probabilities be

$P(E_1) = 1.15$, $P(E_2) = 0.01$, $P(E_3) = P(E_6) = 0.05$

$P(E_4) = 0.50$, $P(E_5) = 0.06$

(a) All event are independent

Using the Boolean algebra approach gives

$$
\begin{aligned}
T &= I_1 + I_2 \\
&= (E_1 I_3) + (E_2 I_4) \\
&= (E_1[E_3 E_4]) + (E_2 + [E_5 + E_6]) \\
&= E_1 E_3 E_4 + E_2 + E_5 + E_6 \qquad (5.14)
\end{aligned}
$$

Substituting for component probabilities and using for combining AND and OR events gives

$P(T) = 0.119245$

Using the numerical approach gives

$P(I_3) = P(E_3)P(E_4) = 0.025$

$P(I_1) = P(E_1)P(I_3) = 0.00375$

$P(I_4) = P(E_5) + P(E_6) – P(E_5)P(E_6) = 0.107$

$P(I_2) = P(E_2) + P(I_4) – P(E_2)P(I_4) = 0.11593$

$P(T) = P(I_1) + P(I_2) – P(I_1)P(I_2) = 0.119245$ (as before)

Both of these methods give exactly the same result. However, it should be noted that basic event E_3 becomes absorbed into intermediate event I_3 using the numerical approach after which its identity is lost. Similarly that of E_6 becomes absorbed into I_4. Since this equivalencing is done before E_3 and E_6 are themselves combined, it is impossible to recognize subsequently that they may well be the same component as in part (b) and hence the same component would be counted twice.

(b) Duplicated events

In this part, it is assumed that E_3 and E_6 are the same component. As noted above, it is virtually impossible to account for this in the numerical approach. However it can be achieved in the Boolean algebra approach by performing a Boolean reduction of the complete statement for T.

Recalling Equation 5.14:

$T = E_1E_3E_4 + E_2 + E_5 + E_6$

Rewriting this by replacing E_6 with E_3 gives

$T = E_1E_3E_4 + E_2 + E_5 + E_3$

$\quad = E_3(E_1E_4 + 1) + E_2 + E_5$

From the rules of Boolean algebra (see Appendix 1):

$$T = E_3 + E_2 + E_5 \qquad (5.15)$$