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Brief Papers

Distributed consensus of multi-agent systems with fault in transmission of control input and time-varying delays

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ABSTRACT

This paper discusses the distributed consensus problem of linear dynamical multi-agent systems with missing control input in some intervals and also delay. At first, assuming zero control input in some intervals and delay, the model of system in such conditions is formulated. Then, a distributed adaptive controller based on the relative states of neighboring agents is proposed. By constructing a set of switching Lyapunov–Krasovskii functional, a new delay-dependent exponential ultimately bounded consensus criterion with explicitly exponential convergence rate is established. Furthermore, the obtained condition will be extended to the multi-agent system, when a false signal is injected instead of the nominal control signal in some intervals. Finally, an illustrative example is solved to show the advantage of the proposed approach.

1. Introduction

A group of autonomous agents can perform with more advantages than a single agent in the dynamical systems. For instance, using a multi-agent system (MAS) leads to

- more adaptivity and scalability of system, and
- more robust against the system and environment faults.

Based on the above reasons, the cooperative control of MAS has been an important research topic in the different areas such as distributed control of vehicles [1–3], flocking and swarming in multi-robot systems [4–6], data fusion and data aggregation in wireless sensor networks [7–10], filtering problems in sensor networks [11], control, filtering and estimation problems in networked control systems [12–15], software MAS in video streaming [16], power systems [17], coordinated defense systems such as synchronization of satellites or spacecraft [18–20]. Among the cooperative problems in MAS, consensus problem means the agreement between agents to reach a common assessment or decision based on the distributed information and a communications protocol. Many of the cooperative problems can be thought of as special cases of consensus; for instance, in formation, when the position of each agent in the geometric pattern is not specified a priori. In general, the consensus problem is an interaction principle among agents that is solved by the control approach.

Recently, numerous results have been reported in the consensus problem of MAS. For instance; in [21–24] the analysis of consensus tracking of continuous-time first and second-order MAS has been studied. In [25–27], consensus tracking for a class of second-order non-linear MAS was studied. Consensus of networks of high-order integrators were studied in [28–31] and linear systems in [32–35,36]. Address a distributed tracking problem for multiple Euler–Lagrange systems. It should be noticed that, considering more general model for MAS can leads to more practical results in this field. As an example, in [31] based on the assumed model, the proposed approach can only be applied to consensus problem of MAS. In this reference, the rendezvous problem could not be solved because of the non-zero velocity in consensus area. In this paper, by selecting the generalized linear model for MAS, rendezvous problem can be solved by the proposed control protocol.

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Also, the existing consensus algorithms reported in various papers can be classified into three classes: consensus without a leader [37], consensus with a leader [38] and model reference consensus [39]. In this work consensus without need to leader will be discussed.

In most applications of MAS, delay is inevitable. Delay in such systems is categorized in two types: communication delay between agents and input delay (delay in the transmission of control input to agents). These can occur due to multi-hop communications, movements of the agents, and unavoidable delays in the communication channels. Delay in such systems usually leads to instability, complexity and hidden oscillations of the system. That is why, the consensus problem in MAS with time-delay has attracted the attention of many researchers; for instance, [40] discussed the consensus problem in directed networks with double-integrator dynamics and non-uniform time-varying communication delays via Lyapunov–Razuminkhin theorem [41], investigated second-order consensus of MAS with time-varying delays based on the Lyapunov stability theory [42], studied the robust consensus problem for higher-order MAS subjected to external disturbance and delays in networks. In [41], second-order group consensus for MAS with constant input delay has been discussed. Asymptotic stability of MAS with topology variances and time-varying delays has been addressed in [43].

On the other hand, fault is an unavoidable phenomenon especially in the complex systems. Fault can result in unsatisfactory performance of the system. It can occur due to provisional failures of communication links, network-induced packet loss, temporary intentional interruption applied to the control block, or false data injection by attacker to the control input in some intervals. These conditions lead to occasional corruption of control signal transmitted to agents which may cause instability and oscillation of agent behavior. These faults often exist in MAS due to various factors as stated above. However, fault tolerant consensus of MAS in such conditions has not been fully discussed. Among the papers in the field of MAS, only in [44] the distributed tracking problem of linear higher-order MAS with occasionally missing control inputs has been studied. The proposed method in [44] depends on eigenvalues of Laplacian matrix. This means results are dependent on global information of the communication graph that may not be available in general.

In summary, the following motivations led to the present work: (i) existence of delay in almost all MAS applications with undesirable effect on the performance of system, (ii) the lack of adequate publications dealing with this issue together with occasionally missing control inputs in MAS, and (iii) work reported in paper [44] is only applicable to system with known communication graph which did not cover the whole area of the current research.

The paper is organized in five sections. In Section 2, the required definitions and lemmas are introduced. In Section 3, the model of system with fault in data transmission from control input to the agents and delay is expressed. In Section 4, the sufficient conditions for the system stability are offered. Finally in Section 5, a simulation example is given to show the advantages of the proposed conditions.

2. Preliminaries

I_N represents the identity matrix of dimension N. 1 Denote a column vector with all entries equal to one. $A \otimes B$ denotes the Kronecker product of matrices A and B. $\|x\|$ denotes its 2-norm. For a symmetric matrix A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote, respectively, the minimum and maximum eigenvalues of A, and the symbol '$*$ ' stands for symmetric blocks in the matrix inequality.

A directed graph G with the set of nodes $V=\{v_1,v_2,\dots,v_n\}$, the set of directed edges $\varepsilon = v\times v$, and a weighted adjacency matrix $A_{adj}=[a_{ij}]_{N\times N}$ with non-negative adjacency elements a_{ij}. An edge e_{ij} in graph G is denoted by the ordered pair of nodes (v_j, v_i), where v_j and v_i are called the parent and child nodes, respectively, and $e_{ij}\in\varepsilon$ if and only if $a_{ij}>0$. Also, the Laplacian matrix $L=[l_{ij}]_{N\times N}$ of G is defined as

$$l_{ij} = \begin{cases} -a_{ij} i\neq j \\ \sum_{k=1,k\neq i}^{N} a_{ik} \quad {}_i i=j \end{cases}$$

Lemma 1. *[1]: Zero is an eigenvalue of L with* 1 *as a right eigenvector and all nonzero eigenvalues have positive real parts. Furthermore, zero is a simple eigenvalue of L, if and only if G has a directed spanning tree.*

Lemma 2. *[45]: For any positive definite matrix $M\in R^{n\times n}$, scalers $\gamma_1<\gamma(t)<\gamma_2$, and a vector matrix $x:\left[-\gamma_2 \quad -\gamma_1\right]\to R^n$ such that the following integrations concerned is well defined, then:*

$$-(\gamma_2-\gamma_1)\left(\int_{t-\gamma_2}^{t-\gamma_1}\dot{x}^T(s)M\dot{x}(s)ds\right) \le -\left[x(t-\gamma_1)-x(t-\gamma_2)\right]^T M\left[x(t-\gamma_1)-x(t-\gamma_2)\right]$$

Lemma 3. *[42]: For any positive definite matrix $R\in R^{n\times n}$ we have*

$$DE+E^TD^T \le DR^{-1}D^T+E^TRE$$

Remark 1. . In this work, it is assumed the communication graph between agents contains a spanning tree. If at least one agent is isolated and do not receive information from other agents, consensus cannot be achieved.

3. Problem statement

In this paper, a network of N agents with linear dynamics is considered. The dynamics of the i-th agent is described by

$$\dot{x}_i = \downarrow Ax_i + A_d x_i(t-d(t)) + Bu_i(t-d(t)),$$
$$x(\theta) = \downarrow \phi(\theta), \theta\in\left[-d_2, 0\right] \quad (1)$$

where $x_i \in R^n$ is the state, $u_i \in R^p$ is the control input, A, A_d, B are the constant known matrices with compatible dimension. $\{\phi(\theta)\}^0_{-d_2}$ is the initial conditions values. $d(t)$ is the time-varying delay which is bounded as

$$d_1 \le d(t) \le d_2, \dot{d}(t) \le \tau < 1$$

In this work we want to solve the consensus problem between agents such that

$$\lim_{t \to \infty}(x_i - x_j) = 0, \ \forall \ i, j = 1, 2, ..., N \tag{2}$$

It is assumed that there is a fault in transmission of control input to agents as follows:

$$u_i = \begin{cases} u_i \ t_{2k} \le t \le t_{2k+1} \\ f_i \ t_{2k+1} < t < t_{2k+2}, \ k \in N \end{cases} \tag{3}$$

Let
$x = [x_1{}^T, x_2{}^T, ..., x_N{}^T]$, $u = [u_1{}^T, u_2{}^T, ..., u_N{}^T]$, $F = \left[f_1{}^T, f_2{}^T, ..., f_N{}^T\right]$ using (1), (3) the following closed loop system is obtained

$$\dot{x}(t) = \begin{cases} ((I_N \otimes A)x(t) + (I_N \otimes A_d)x(t-d(t)) + (I \otimes B)u \ t_{2k} \le t \le t_{2k+1} \\ (I_N \otimes A)x(t) + (I_N \otimes A_d)x(t-d(t)) + (I \otimes B)F \ t_{2k+1} < t < t_{2k+2}, \ k \in N \end{cases} \tag{4}$$

Based on (2), the disagreement between agents can be expressed in the following form:

$$\zeta_i = x_i - 1/N \sum_{j=1}^{N} x_j(t), \tag{5}$$

which can be presented in the following vector form:

$$\zeta = (M \otimes I_n)x, \ M = I_N - 1/N11^T, \tag{6}$$

From (4) and (6) the following equation for consensus error is obtained:

$$\dot{\zeta}(t) = \begin{cases} (I_N \otimes A)\zeta + (I_N \otimes A_d)\zeta(t-d(t)) + (M \otimes B)u \ t_k \le t \le t_{2k+1} \\ (I_N \otimes A)\zeta + (I_N \otimes A_d)\zeta(t-d(t)) + (M \otimes B)F \ t_{2k+1} < t < t_{2k+2}, \ k \in N \end{cases} \tag{7}$$

4. Main results

4.1. Missing control input in some intervals

In this section, we analysis the delay dependent consensus condition for MAS (1) with fault in the control input as (3) with $F = 0$.

To achieve distributed consensus and simultaneously independence of controller gain from eigenvalues of Laplacian matrix, the following controller is suggested:

$$u_i = c_i K \sum_{j=1}^{N} a_{ij}(x_j - x_i),$$

$$\dot{c}_i = \eta_i \left[-\gamma_i c_i + \left(\sum_{j=1}^{N} a_{ij}(x_i - x_j)^T \Lambda \sum_{j=1}^{N} a_{ij}(x_i(t-d(t)) - x_j(t-d(t)) \right) \right], \tag{8}$$

where $K \in R^{n \times p}$, η_i, γ_i are positive scalars and $\Lambda = -P_1 BK$.

For the stability analysis of system (7) with controller (8), the stability of system in cases of non-failure and failure of the controller is analyzed, separately.

At first, the stability analysis of system during $t_k \le t \le t_{2k+1}$ is investigated. We choose the following Lyapunov–Krasovskii functional

$$V_1 = V_{11} + V_{12} + V_{13} \tag{9}$$

With

$$V_{11} = 1/2\zeta^T(t)(L \otimes P_1)\zeta(t) + \sum_{i=1}^{N} \frac{\bar{c}_i{}^2}{2\eta_i},$$

$$V_{12} = \int_{t-d(t)}^{t} e^{\alpha_1(s-t)}\zeta^T(s)(L \otimes Q_1)\zeta(s)ds,$$

$$V_{13} = \int_{-d(t)}^{0}\int_{t+\theta}^{t} e^{\alpha_1(s-t)}\dot{\zeta}^T(s)(L \otimes R_1)\dot{\zeta}(s)dsd\theta,$$

where $\bar{c}_i = c_i - \widehat{c}, \hat{c} = \frac{1}{\lambda_2}$ and λ_2 denotes the smallest nonzero eigenvalue of L. Also, we define $C = diag(c_i, i = 1, ..., N)$.

Now, we compute the time-derivative of $V1(t)$

$$\dot{V}_{11} = 1/2\zeta^T(t)(L \otimes P_1 A)\zeta(t) + 1/2\zeta^T(t)(L \otimes P_1 A_d)\zeta(t-d(t)) + 1/2\zeta^T(t)(L \otimes A^T P_1)\zeta(t) + 1/2\zeta^T(t-d(t))(L \otimes A_d{}^T P_1)\zeta(t) + \zeta^T(t)(CL^2 \otimes P_1 BK)\zeta(t-d(t))$$
$$+ \sum_{j=1}^{N} \bar{c}_i \left[-\gamma_i c_i + \left(\sum_{j=1}^{N} a_{ij}(x_i - x_j)^T \Lambda \sum_{j=1}^{N} a_{ij}(x_i(t-d(t)) - x_j(t-d(t)) \right) \right], \tag{10}$$

$$\dot{V}_{12}=\zeta^T(t)(L\otimes Q_1)\zeta(t)-e^{-\alpha_1 d(t)}(1-\tau)\zeta^T(t-d(t))(L\otimes Q_1)\zeta(t-d(t))-\alpha_1 V_{12}\le \zeta^T(t)(L\otimes Q_1)\zeta(t)-e^{-\alpha_1 d_2}(1-\tau)\zeta^T(t-d(t))(L\otimes Q_1)\zeta(t-d(t)) -\alpha_1 V_{12}, \tag{11}$$

$$\begin{aligned}\dot{V}_{13}&=d(t)\dot{\zeta}^T(t)(L\otimes R_1)\dot{\zeta}(t)-d(t)\int_{t-d(t)}^{t}e^{\alpha_1(s-t)}\dot{\zeta}^T(t)(L\otimes R_1)\dot{\zeta}(t)ds-\alpha_1 V_{13}\\ &\le d_2\dot{\zeta}^T(t)(L\otimes R_1)\dot{\zeta}(t)-d(t)e^{-\alpha_1 d_2}\int_{t-d(t)}^{t}\dot{\zeta}^T(t)(L\otimes R_1)\dot{\zeta}(t)ds-\alpha_1 V_{13},\end{aligned} \tag{12}$$

In addition

$$\zeta^T(t)(CL^2\otimes P_1BK)\zeta(t-d(t))=-\sum_{j=1}^{N}(\overline{c}_i+\widehat{c})\left(\sum_{j=1}^{N}a_{ij}(x_i-x_j)^T\Lambda\sum_{j=1}^{N}a_{ij}(x_i(t-d(t))-x_j(t-d(t))\right), \tag{13}$$

So

$$\dot{V}_{11}=\Pi-\sum_{j=1}^{N}(\overline{c}_i+\widehat{c})a_{ij}(x_i-x_j)^T\Lambda\sum_{j=1}^{N}a_{ij}(x_i(t-d(t))-x_j(t-d(t)))+\sum_{i=1}^{N}-\gamma_i c_i\overline{c}_i+\sum_{i=1}^{N}\overline{c}_i\left(\sum_{j=1}^{N}a_{ij}(x_i-x_j)^T\Lambda\sum_{j=1}^{N}a_{ij}(x_i(t-d(t))-x_j(t-d(t))\right), \tag{14}$$

With

$$\Pi=1/2\zeta^T(t)(L\otimes P_1A)\zeta(t)+1/2\zeta^T(t)(L\otimes P_1A_d)\zeta(t-d(t))+1/2\zeta^T(t)(L\otimes A^TP_1)\zeta(t)+1/2\zeta^T(t-d(t))(L\otimes A_d{}^TP_1)\zeta(t)$$

which results

$$\dot{V}_{11}=\Pi+\sum_{j=1}^{N}-\gamma_i c_i\overline{c}_i+\zeta^T(t)(\hat{c}L^2\otimes P_1BK)\zeta(t-d(t)) \tag{15}$$

Let's define $U=\begin{bmatrix}1/\sqrt{N} & Y_1\end{bmatrix}, U^T=\begin{bmatrix}1/\sqrt{N}\\ Y_2\end{bmatrix}$, such that $U^TLU=diag\{0,\lambda_2,...,\lambda_N\}$ and $\overline{\zeta}=(U^T\otimes I)\zeta$,

Therefore

$$\begin{aligned}\dot{V}_1+\alpha_1 V_1\le &-\sum_{i=1}^{N}\gamma_i c_i\overline{c}_i+\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}+\Pi+\alpha_1\zeta^T(t)(L\otimes P_1)\zeta(t)+\zeta^T(t)(\hat{c}L^2\otimes P_1BK)\zeta(t-d(t))\\ &+\zeta^T(t)(L\otimes Q_1)\zeta(t)-e^{-\alpha_1 d_2}(1-\tau)\zeta^T(t)(t-d(t))(L\otimes Q_1)\zeta(t-d(t))+d_2\dot{\zeta}^T(t)(L\otimes R_1)\dot{\zeta}(t)\\ &-d(t)e^{-\alpha_1 d_2}\int_{t-d(t)}^{t}\dot{\zeta}^T(t)(L\otimes R_1)\dot{\zeta}(t)ds=-\sum_{i=1}^{N}\gamma_i c_i\overline{c}_i+\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}+\sum_{i=2}^{N}\lambda_i(\overline{\zeta}_i{}^T(t)(1/2A^TP_1+1/2P_1A+\alpha_1P_1+Q_1)\overline{\zeta}_i(t)\\ &+\overline{\zeta}_i{}^T(t)(1/2A_d{}^TP_1+1/2P_1A_d+\widehat{c}\lambda_iP_1BK)\overline{\zeta}_i(t-d(t))-\overline{\zeta}_i{}^T(t-d(t))e^{-\alpha_1 d_2}(1-\tau)Q_1\overline{\zeta}_i(t-d(t)))+d_2\dot{\zeta}^T(t)(L\otimes R_1)\dot{\zeta}(t)\\ &-d(t)e^{-\alpha_1 d_2}\int_{t-d(t)}^{t}\dot{\zeta}^T(t)(L\otimes R_1)\dot{\zeta}(t)ds\end{aligned}$$

$$\begin{aligned}=&-\sum_{i=1}^{N}\gamma_i c_i\overline{c}_i+\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}+\sum_{i=2}^{N}\lambda_i(\overline{\zeta}_i{}^T(t)(1/2A^TP_1+1/2P_1A+\alpha_1P_1+Q_1)\overline{\zeta}_i(t)+\overline{\zeta}_i{}^T(t)(1/2A_d{}^TP_1+1/2P_1A_d+P_1BK)\overline{\zeta}_i(t-d(t))\\ &-\overline{\zeta}_i{}^T(t-d(t))e^{-\alpha_1 d_2}(1-\tau)Q_1\overline{\zeta}_i(t-d(t)))+d_2\dot{\overline{\zeta}}^T(t)(L\otimes R_1)\dot{\overline{\zeta}}(t)-d(t)e^{-\alpha_1 d_2}\int_{t-d(t)}^{t}\dot{\overline{\zeta}}^T(t)(L\otimes R_1)\dot{\overline{\zeta}}(t)ds,\end{aligned} \tag{16}$$

Hence

$$\begin{aligned}\dot{V}_1+\alpha_1 V_1\le &-\sum_{i=1}^{N}\gamma_i/2\hat{c}^2+\sum_{i=2}^{N}\lambda_i(\overline{\zeta}_i{}^T(t)(1/2A^TP_1+1/2P_1A+\alpha_1P_1+Q_1)\overline{\zeta}_i(t)+\overline{\zeta}_i{}^T(t)(1/2A_d{}^TP_1+1/2P_1A_d+P_1BK)\overline{\zeta}_i(t-d(t))\\ &+\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}-\overline{\zeta}_i{}^T(t-d(t))e^{-\alpha_1 d_2}(1-\tau)Q_1\overline{\zeta}_i(t-d(t))+d_2\dot{\overline{\zeta}}_i{}^T(t)R_1\dot{\overline{\zeta}}_i(t)-d(t)e^{-\alpha_1 d_2}\int_{t-d(t)}^{t}\dot{\overline{\zeta}}_i{}^T(t)R_1\dot{\overline{\zeta}}_i(t)ds).\end{aligned} \tag{17}$$

Using Lemma 2, the following condition is achieved:

$$-d(t)e^{-\alpha_1 d_2}\int_{t-d(t)}^{t}\dot{\overline{\zeta}}_i{}^T(t)R_1\dot{\overline{\zeta}}_i(t)ds\le -e^{-\alpha_1 d_2}(\overline{\zeta}_i(t)-\overline{\zeta}_i(t-d(t))^TR_1(\overline{\zeta}_i(t)-\overline{\zeta}_i(t-d(t)) \tag{18}$$

Finally, from (17) and (18), applying Lemma 3 and Schur Complement, if the following inequality is satisfied:

$$\begin{bmatrix}1/2A^TP_1+1/2P_1A+\alpha_1P_1+Q_1-e^{-\alpha_1 d_2}R_1 & 1/2A_d{}^TP_1+1/2P_1A_d+e^{-\alpha_1 d_2}R_1+d_2A_d{}^TR_1A & A^T & A_d{}^T & K^TB^T & d_2K^TB^T & A^T\\ * & -e^{-\alpha_1 d_2}(1-\tau)Q_1{}^T-e^{-\alpha_1 d_2}R_1 & 0 & 0 & 0 & 0 & 0\\ * & * & -1/d_2R_1{}^{-1} & 0 & 0 & 0 & 0\\ * & * & * & -1/d_2R_1{}^{-1} & 0 & 0 & 0\\ * & * & * & * & -1/d_2R_1{}^{-1} & 0 & 0\\ * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & -R_1{}^{-1}\end{bmatrix}<0 \tag{19}$$

We have

$$\dot{V}_1+\alpha_1 V_1<\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}-\sum_{i=1}^{N}\gamma_i/2\hat{c}^2, \tag{20}$$

or

$$V_1<(V_1(\phi)-\frac{1}{\alpha_1}\left(\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}-\sum_{i=1}^{N}\gamma/2\hat{c}_i\hat{c}_i)\right)e^{-\alpha_1 t}+\frac{1}{\alpha_1}\left(\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}-\sum_{j=1}^{N}\gamma/2\hat{c}_i\hat{c}_i\right). \tag{21}$$

From (20), it can be concluded that system (7) with controller (8) is exponentially convergence with a rate less than α_1. We assume this rate is μ_1. Hence, (20) can be rewritten as

$$\dot{V}_1<-\mu_1 V_1-(\alpha_1-\mu_1)V_1-\sum_{i=1}^{N}\gamma_i/2\hat{c}^2+\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}, \tag{22}$$

On the other hand, based on the defined Lyaponov–Krasovskii functional, we have

$$1/2\lambda_2\lambda_{\min}(P_1)\|\zeta\|^2\le V_1 \tag{23}$$

Hence

$$\dot{V}_1<-\mu_1 V_1-\frac{(\alpha_1-\mu_1)}{2}\lambda_2\lambda_{\min}(P_1)\|\zeta\|^2-\sum_{i=1}^{N}\gamma_i/2\hat{c}^2+\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}, \tag{24}$$

Therefore, if

$$\|\zeta\|^2<\frac{2}{\lambda_2\lambda_{\min}(P_1)(\alpha_1-\mu_1)}\cdot\left(-\sum_{i=1}^{N}\gamma_i/2\hat{c}^2+\alpha_1\sum_{i=1}^{N}\frac{\overline{c}_i{}^2}{2\eta_i}\right) \tag{25}$$

We reach

$$\dot{V}_1<-\mu_1 V_1\square. \tag{26}$$

Now, for the stability analysis of system in $t_{k+1}\le t\le t_{2k+2}$, we choose the following Lyapunov–Krasovskii functional:

$$V_2=V_{21}+V_{22}+V_{23}, \tag{27}$$

where

$$V_{21}=\zeta^T(t)(L\otimes P_2)\zeta(t)$$

$$V_{22}=\int_{t-d(t)}^{t}e^{-\alpha_2(s-t)}\zeta^T(s)(L\otimes Q_2)\zeta(s)ds$$

$$V_{23}=\int_{-d(t)}^{0}\int_{t+\theta}^{t}e^{-\alpha_2(s-t)}\dot{\zeta}^T(s)(L\otimes R_2)\dot{\zeta}(s)dsd\theta$$

Now we compute the time-derivative of $V_2(t)$

$$\dot{V}_{21}=1/2\zeta^T(t)(L\otimes P_2A)\zeta(t)+1/2\zeta^T(t)(L\otimes P_2A_d)\zeta(t-d(t))+1/2\zeta^T(t)(L\otimes A^TP_2)\zeta(t)+1/2\zeta^T(t-d(t))(L\otimes A_d{}^TP_2)\zeta(t) \tag{28}$$

$$\dot{V}_{22}\le\zeta^T(t)(L\otimes Q_2)\zeta(t)-e^{\alpha_2 d_1}(1-\tau)\zeta^T(t-d(t))(L\otimes Q_2)\times\zeta(t-d(t))+\alpha_2 V_{22}, \tag{29}$$

$$\dot{V}_{23}\le d_2\dot{\zeta}^T(t)(L\otimes R_2)\dot{\zeta}(t)-d(t)e^{\alpha_2 d_1}\int_{t-d(t)}^{t}\dot{\zeta}^T(t)(L\otimes R_2)\dot{\zeta}(t)ds+\alpha_2 V_{23}, \tag{30}$$

Following the same steps of previous, if the following inequality is satisfied:

$$\begin{bmatrix}1/2A^TP_2+1/2P_2A-\alpha_2P_2+Q_2-e^{\alpha_2 d_1}R_2+d_2A^TR_2A & 1/2A_d{}^TP_2+1/2P_2A_d+e^{\alpha_2 d_1}R_2+d_2A_d{}^TR_2A\\ * & -e^{\alpha_2 d_1}(1-\tau)Q_2{}^T-e^{\alpha_2 d_1}R_2+d_2A_d{}^TR_2A_d\end{bmatrix}<0 \tag{31}$$

We have

$$V_2<(V_2(\phi))e^{\alpha_2 t}\square. \tag{32}$$

The following theorem shows that the consensus of system (4) with $F=0$ can be guaranteed if there exist some matrices satisfying certain matrix inequalities.

Theorem 1. . *Consensus in MAS (4) and $F=0$ with control input (8) is satisfied for any time-varying delay $d(t)$ satisfying $d_1\le d(t)\le d_2$, if the following conditions are met:*

1. *There exist positive define matrices P_1,Q_1,R_1,P_2,Q_2,R_2, such that the matrix inequalities (19) and (31) are satisfied.*
2. *There is $\rho>1$ such that*

$$(P_1,Q_1,R_1)>\rho(P_2,Q_2,R_2),.$$

3. *There is* $\delta > 0$ *such that*
$V_1 \le (\rho + \delta)V_2$
4. *If* N_{fail} *and* T_{fail} *are defined the number of failures and the total time of failures of input signal in* $t \in [t_1, t_2]$; *there exist* $\alpha^* > \alpha > 0$ *such that*
$-\mu_1(t - T_{fail}) + \alpha_2(T_{fail}) \le \alpha^* t, (\rho + \delta)^{N_{fail}(t)} \le e^{\alpha t}$.

Proof. for proof we select the following switching Lyapunov–Krasovskii functional:

$$V(t) \le \begin{cases} V_1(t) \text{ if } t_{2k} \le t < t_{2k-1} \\ V_2(t) \text{ if } t_{2k+1} \le t < t_{2k+2} \end{cases}, \tag{33}$$

From inequality (26), (32) we have:

$$V(t) \le \begin{cases} e^{-\mu_1(t-t_{2k})} V_1(t_{2k}) \text{ if } t_{2k} \le t < t_{2k-1} \\ e^{\alpha_2(t-t_{2k})} V_2(t_{2k+1}) \text{ if } t_{2k+1} \le t < t_{2k+2} \end{cases}, \tag{34}$$

For $t_0 \in [t_{2k}, t_{2k+1}]$

$$\begin{aligned} V(t) &\le e^{-\mu_1(t-t_{2k})} V1(t_{2k}) \le e^{-\mu_1(t-t_{2k})} V2(t_{2k^-}) \le e^{-\mu_1(t-t_{2k})}(\rho+\delta)(e^{\alpha 2(t_{2k}-t_{2k-1})} V2(t_{2k-1})) \le e^{-\mu_1(t-t_{2k})}(\rho+\delta)(e^{\alpha 2(t_{2k}-t_{2k-1})} V1(t_{2k-1^-})) \\ &\le e^{-\mu_1(t-t_{2k})}(\rho+\delta)^2 e^{\alpha 2(t_{2k}-t_{2k-1})}(e^{-\mu_1(t_{2k-1}-t_{2k-2})} V1(t_{2k-2}))) \le \ldots \le (\rho+\delta)^k e^{-\alpha_1(t-T_{fail}(t))} e^{\alpha 2(T_{fail}(t))} V(\phi), \end{aligned} \tag{35}$$

And for $t_0 \in [t_{2k+1}, t_{2k+2}]$ we have

$$V(t) \le e^{\alpha_2(t-t_{2k+1})} V2(t_{2k+1}) \le e^{\alpha_2(t-t_{2k+1})} V1(t_{2k+1^-}) \le e^{\alpha_2(t-t_{2k+1})}((e^{-\mu_1(t_{2k+1}-t_{2k})} V1(t_{2k})) \le \ldots \le (\rho+\delta)^{k+1} e^{-\mu_1(t-T_{fail}(t))} e^{\alpha_2(T_{fail}(t))} V(\phi) \tag{36}$$

Finally, from (35) and (36) the proof is completed and we have:

$$V(t) \le e^{-(\alpha*-\alpha)t} V(\phi) \square. \tag{37}$$

Remark 2. . It is noted that inequality (19) provide stability criterion for the MAS (1). It depends on the bounds of the time-varying delay of MAS. Hence, this result is less conservative compared to results in [40]. Moreover, our result is more favorable than other available publications such as [42,43]. Here, exponential stability is guaranteed while some previous works such as [42,43] has discussed on asymptotic stability of MAS. This enables one to check the exponential stability of MAS with a prescribed decay rate. Such criterion is more practical in control field, because a fast response for a controlled system is often needed. This is especially true in controller failure cases that may exist only a short availability controller time.

Remark 3. : In this section, we select a Lyapunov–Krasovskii functional with three terms. Generally, the main reason for this selection is to choose a functional with minimum terms to simultaneously achieve less conservatism and minimal computational time. The first objective is implemented through generating conditions that are dependent on the bounds of the time-varying delay of system. The second one is achieved by minimizing the variables included in the functional to reduce the computation load. The latter objective is of paramount importance for fault tolerant system with occasional failures, because the control signal availability might exist for only short periods of time. This will limit the time allocated to controller gain computations and hence an efficient algorithm is necessary. This subject calls for further research.

4.2. *Injected false signal to control input in some intervals*

Now, the assumption in Section 4.1 is further extended to include a non-zero fault value for each input. Hence, generalizing the previous result, it is assumed that the fault function F in (3) has the following constrain:

$$\|f_i(x_i, t)\| \le \bar{e}_i \|x_i\|, i = 1, 2, \ldots N. \tag{38}$$

where $\bar{e}_i$ is unknown. To control the system in such condition, we apply the following controller:

$$u_i = c_i K \sum_{j=1}^{N} a_{ij}(x_j - x_i) + \Theta \tag{39}$$

with

$$\dot{c}_i = \eta_i \left[-\gamma_i c_i + \left(\sum_{j=1}^{N} a_{ij}(x_i - x_j)^T \Lambda \sum_{j=1}^{N} a_{ij}(x_i(t-d(t)) - x_j(t-d(t)) \right) \right]$$

$$\dot{e}_i = \varepsilon_i \left[-\kappa_i e_i + \|K \sum_{j=1}^{N} a_{ij}(x_i - x_j)\| \|x_i\| \right]$$

$$\Theta = -e_i \|x_i\|,$$

where ε_i, κ_i are positive scalars.

Similar to previous section, for the stability analysis of system in $t_k \le t \le t_{2k+1}$, we choose the following Lyapunov–Krasovskii functional:

$$V_3 = V_{31} + V_{32} + V_{33} \tag{40}$$

with

$$V_{31} = 1/2\zeta(t)^T (L \otimes P_3)\zeta(t) + \sum_{i=1}^{N} \frac{\tilde{c}_i^2}{2\eta_i} + \sum_{i=1}^{N} \frac{\tilde{e}_i^2}{2\varepsilon_i}, V_{32} = \int_{t-d(t)}^{t} e^{\alpha_3(s-t)} \zeta^T(s)(L \otimes Q_3)\zeta(s) ds V_{33} = \int_{-d(t)}^{0} \int_{t+\theta}^{t} e^{\alpha_3(s-t)} \dot{\zeta}^T(s)(L \otimes R_3)\dot{\zeta}(s) ds d\theta$$

where $\tilde{e}_i = e_i - \bar{e}_i$.

Now we compute the time-derivative of V_3 :

$$\dot{V}_{31} = \dot{V}_{11}(P_3, Q_3, R_3) + \zeta^T (L \otimes P_3 B)\Theta + \sum_{j=1}^{N} \tilde{e}_i \left[-\kappa_i e_i + \left\| K \sum_{j=1}^{N} a_{ij}(x_i - x_j) \right\| \|x_i\| \right], \tag{41}$$

We have

$$\zeta^T (L \otimes P_3 B)\Theta = -\sum_{j=1}^{N} e_i \|x_i\| \left\| B^T P \sum_{j=1}^{N} l_{ij}\zeta_j \right\| \tag{42}$$

Hence

$$\dot{V}_{31} = \dot{V}_{11}(P_3, Q_3, R_3) + \sum_{j=1}^{N} \tilde{e}_i \left[-\kappa_i e_i + \left\| \sum_{j=1}^{N} a_{ij}(x_i - x_j) \right\| \|x_i\| \right] - \sum_{j=1}^{N} e_i \|x_i \| \left\| B^T P \sum_{j=1}^{N} l_{ij}\zeta_j \right\| \tag{43}$$

On the other hand:

$$\sum_{j=1}^{N} \tilde{e}_i \left[-\kappa_i e_i + \left\| \sum_{j=1}^{N} a_{ij}(x_i - x_j) \right\| \|x_i\| \right] - \sum_{j=1}^{N} e_i \|x_i\| \left\| B^T P \sum_{j=1}^{N} l_{ij}\zeta_j \right\| = \sum_{j=1}^{N} -\kappa_i e_i \tilde{e}_i - e_i \left\| \sum_{j=1}^{N} a_{ij}(x_i - x_j) \right\| \|x_i\|. \tag{44}$$

We define $\lambda = \bar{e}_i \left\| \sum_{j=1}^{N} a_{ij}(x_i - x_j) \right\| \|x_i\|$.

Hence, if the following inequality is satisfied:

$$\begin{bmatrix} 1/2A^T P_3 + 1/2P_3 A + \alpha_3 P_3 + Q_3 - e^{-\alpha_3 d_2} R_3 & 1/2A_d{}^T P_3 + 1/2P_3 A_d + e^{-\alpha_3 d_2} R_3 + d_2 A_d{}^T R_3 A & A^T & A_d{}^T & K^T B^T & d_2 K^T B^T & A^T \\ * & -e^{-\alpha_3 d_2}(1-\tau)Q_3{}^T - e^{-\alpha_3 d_2} R_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & -1/d_2 R_3{}^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & -1/d_2 R_3{}^{-1} & 0 & 0 & 0 \\ * & * & * & * & -1/d_2 R_3{}^{-1} & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -R_3{}^{-1} \end{bmatrix} < 0 \tag{45}$$

We reach:

$$\dot{V}_3 + \alpha_3 V_3 < -\sum_{i=1}^{N} \gamma_i / 2\hat{c}^2 + \alpha_3 \sum_{i=1}^{N} \frac{\bar{c}_i{}^2}{2\eta_i} + \sum_{i=1}^{N} \kappa_i / 2\tilde{e}^2 - \lambda, \tag{46}$$

And similar to previous analysis (22)–(24)), for:

$$\|\zeta\|^2 < \frac{2}{\lambda_2 \lambda_{\min}(P_3)(\alpha_1 - \mu_2)} \cdot \sum_{i=1}^{N} \left(-\gamma/2\hat{c}^2 + \alpha_3 \sum_{i=1}^{N} \frac{\bar{c}_i{}^2}{2\eta_i} + \gamma/2\tilde{e}^2 - \lambda \right) \tag{47}$$

We have:

$$\dot{V}_3 < -\mu_2 V_3, \mu_2 < \alpha_3 \square. \tag{48}$$

Now, for the stability analysis of system in $t_{k+1} \le t \le t_{2k+2}$, we choose the following Lyapunov–Krasovskii functional:

$$V_4 = V_{41} + V_{42} + V_{43} \tag{49}$$

where

$$V_{41} = 1/2\zeta(t)^T (L \otimes P_4)\zeta(t) \; V_{42} = \int_{t-d(t)}^{t} e^{-\alpha_4(s-t)} \zeta^T(s)(L \otimes Q_4)\zeta(s) ds \; V_{43} = \int_{-d(t)}^{0} \int_{t+\theta}^{t} e^{-\alpha_4(s-t)} \dot{\zeta}^T(s)(L \otimes R_4)\dot{\zeta}(s) ds d\theta$$

Now we compute the time-derivative of $V_4(t)$:

$$\dot{V}_{41} = \dot{V}_{21} + \zeta^T (L \otimes P_4 B)F, \tag{50}$$

We have:

$$\zeta^T (L \otimes P_4 B)F \le \sum_{j=1}^{N} \bar{e}_i \|x_i\| \left\| B^T P \sum_{j=1}^{N} l_{ij}\zeta_j \right\| \tag{51}$$

Hence:

$$\dot{V}_{41} \le \dot{V}_{21} + \sum_{j=1}^{N} \bar{e}_i \|x_i\| \left\| B^T P_4 \sum_{j=1}^{N} l_{ij}\zeta_j \right\| \tag{52}$$

If the following inequality is satisfied:

$$\begin{bmatrix} 1/2A^TP_4+1/2P_4A-\alpha_4P_4+Q_4-e^{\alpha_4 d_1}R_4+d_2A^TR_4A & 1/2{A_d}^TP_4+1/2P_4A_d+e^{\alpha_4 d_2}R_4+d_2{A_d}^TR_4A \\ * & -e^{\alpha_4 d_1}(1-\tau){Q_4}^T-e^{\alpha_4 d_1}R_4+d_2{A_d}^TR_4A_d \end{bmatrix}<0 \tag{53}$$

We reach:

$$\dot{V}_4-\alpha_4V_4<\lambda, \tag{54}$$

And finally for:

$$\|\zeta\|^2<\frac{2}{\lambda_2\lambda_{\min}(P_4)(-\alpha_4+\mu_3)}.(\lambda),\mu_3<\alpha_4 \tag{55}$$

we have:

$$\dot{V}_4<\mu_3V_4\square. \tag{56}$$

The following theorem shows that the consensus of system (4) with fault as (38) can be guaranteed if there exist some matrices satisfying certain matrix inequalities.

Theorem 2. *. Consensus in MAS (4) and fault as (38) with control input (39) is satisfied for any time-varying delay $d(t)$ satisfying $d_1\le d(t)\le d_2$, if the following conditions are met:*

1. *There exist positive define matrices P_3,Q_3,R_3,P_4,Q_4,R_4 such that the matrix inequalities (45), (53) are satisfied.*
2. *There is $\rho>1$ such that*

   $(P_3,Q_3,R_3)>\rho(P_4,Q_4,R_4),.$

3. *There is $\delta>0$ such that*
   $V_3\le(\rho+\delta)V_4$
4. *If we defineN_{fail} the number of failures andT_{fail} the total time of failures of input signal in $t\in[t_1,t_2]$, there exist $\alpha^*>\alpha>0$, such that*

   $-\mu_2(t-T_{fail})+\mu_3(T_{fail})\le\alpha^*t,(\rho+\delta)^{N_{fail}(t)}\le e^{\alpha t}.$

Proof. Similar to previous proof and (45), (53).

5. Numerical simulation

In this section, a numerical example is given to verify our proposed Theorems. Note that this example has been discussed in [44]; however, we solve it including our stated constrains Fig. 1.

Example. Consider a network of linear MAS as follows:
$$\dot{x}_i=Ax_i+A_dx_i(t-d(t))+Bu_i(t-d(t)),i=1,2,...,7 \tag{57}$$

With:

$$A=\begin{bmatrix} 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1\\ 0&0&-0.2003&-0.2003&0&0\\ 0&0&0.2003&0&-0.2003&0\\ 0&0&0&0&0&-1.6129 \end{bmatrix},Ad=\begin{bmatrix} -1&0&0&-0.01&0&0\\ 0&0&0&0&0.01&0\\ 0&0&0&0&0.01&0\\ 0&0&0.0003&0.0003&0&0\\ 0&0&-0.0001&0&-0.0003&0\\ 0&0&0&0&0&-0.005 \end{bmatrix},$$

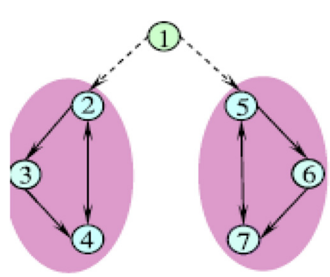

Fig. 1. Communication graph of system (57).

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.9441 & 0.9441 \\ 0.9441 & 0.9441 \\ -28.7097 & 28.7097 \end{bmatrix}, d(t) = 0.3|\sin(t)|.$$

In this example, it is assumed that because of the controller failures, for instance packet loss of input signal in 0.15 of time intervals, the control signal is zero until a new packet arrives. To reach the consensus in such condition, we use our obtained result in Theorem 1. At first, we assume ρ, α_1, α_2 respectively as 1.021, 0.33, 0.32 to solve inequalities in (19) and (31). Applying free weighting matrix (FWM) algorithm for solving the matrix inequality in (19) with three unknown variables, the gain of controller (8) for satisfying Theorem 1 is

$$K = \begin{bmatrix} 6.8017 & -6.146 & -0.65543 & 6.81646 & -5.279 & -0.07198 \\ 6.146 & 11.8567 & 0.6554 & -5.2479 & 6.81646 & 0.07198 \end{bmatrix},$$

Also, using Matlab LMI Toolbox, we can conclude that the obtained LMI in (31) with selected decay rate is feasible. Hence, by Theorem 1, it can be deduced that the consensus problem for system (57) can be solved. Result of the simulation of closed-loop system (Figs. 2–7) shows that we could solve the consensus problem in this example with our proposed controller. It should be noticed according to Theorem 1 selecting $\alpha = 0.12, \alpha^* = 0.23$ the maximum value of $T_{fail}/t_2 - t_1$ with the above assumption is 0.27 which determines the allowable time limit of packet loss. Also, with these values of exponential rates, the maximum value for the upper bound of system delay and maximum number of controller failures in this interval are 0.471 and 6.17 respectively.

Now we want to evaluate our result in Theorem 2. In this case, we assume the system (57) with a change that in 0.15 of time intervals the false input signal is injected instead of the true input signal by attacker until the true data arrives. This fault is presented as follows:

$$f(x_i) = 0.34 \ \sin(x_i), \tag{58}$$

Assuming ρ, α_1, α_2 respectively as 1.021, 0.33, 0.32, and following the same approach as above, the gain of controller in (39) for satisfying Theorem 2 is

$$K = \begin{bmatrix} 8.529 & 12.16 & -1.2966 & 13.4846 & -10.3818 & -0.1424 \\ 25.800 & 1.3587 & 0.323 & -11.0157 & 14.828 & 0.376 \end{bmatrix},$$

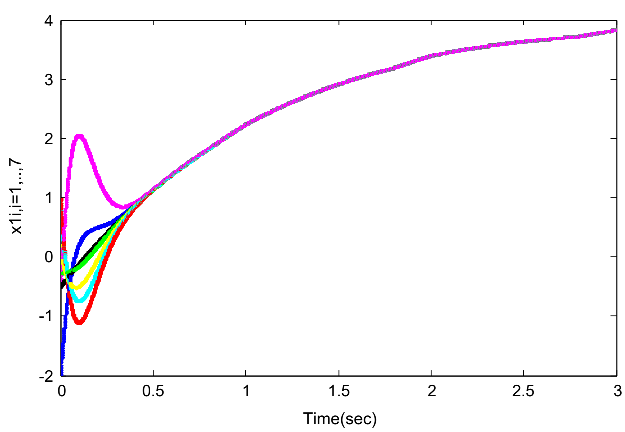

Fig. 2. The behaviors of the $x_{1i}, i = 1, ..., 7$ in the controlled system (57).

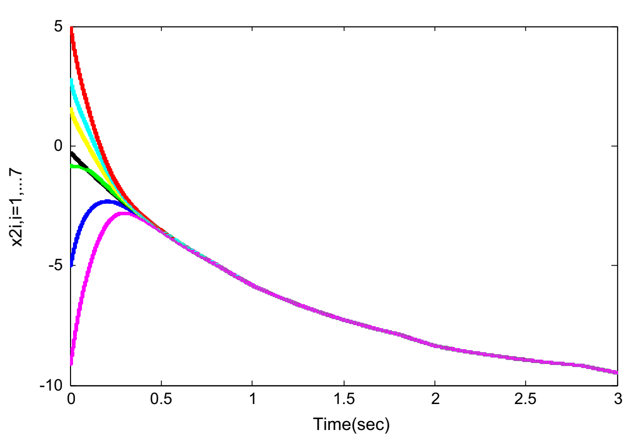

Fig. 3. The behaviors of the $x_{2i}, i = 1, ..., 7$ in the controlled system (57).

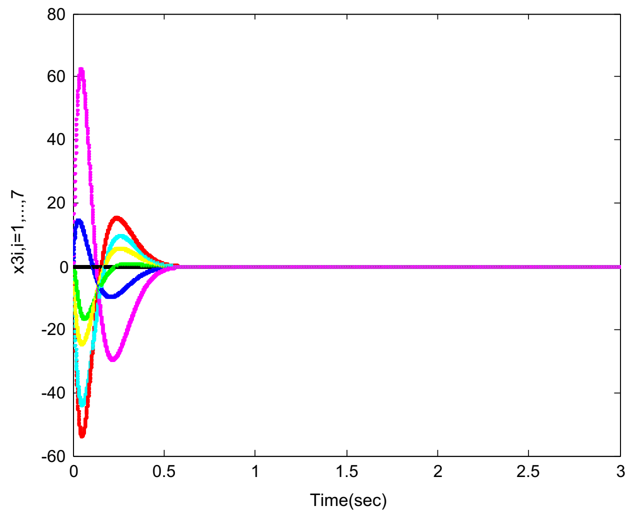

Fig. 4. The behaviors of the $x_{3i}, i = 1, \ldots, 7$ in the controlled system (57).

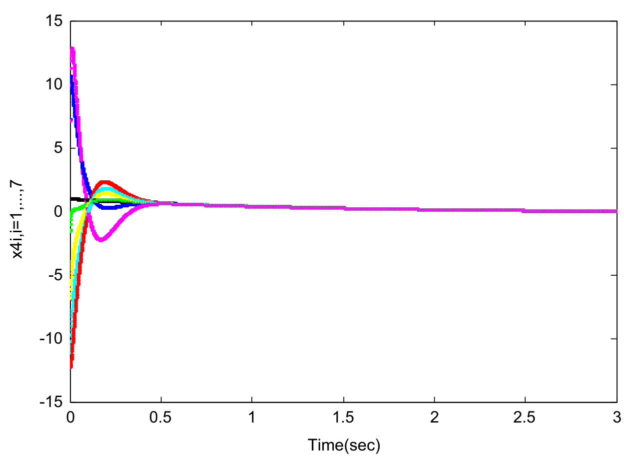

Fig. 5. The behaviors of the $x_{4i}, i = 1, \ldots, 7$ in the controlled system (57).

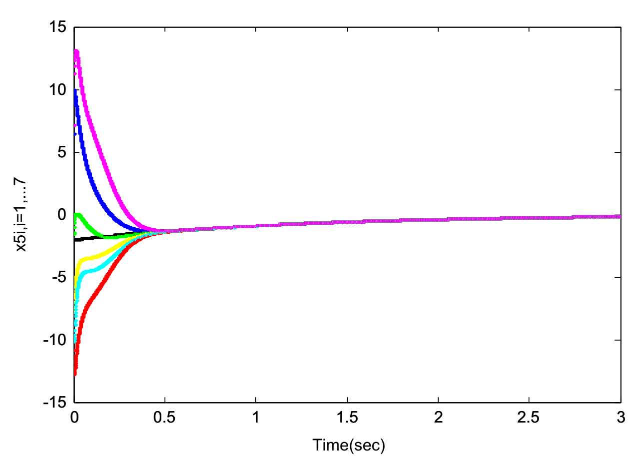

Fig. 6. The behaviors of the $x_{5i}, i = 1, \ldots, 7$ in the controlled system (57).

Result of the simulation of closed-loop system (Figs. 8–13) shows that we could solve the consensus problem between agents in this case with our proposed controller. Similar to previous discuss for packetloss, it should be noticed that according to Theorem 2 selecting $\alpha = 0.12, \alpha^* = 0.23$ the maximum value of $T_{fail}/t_2 - t_1$ with the above assumption for system (57) with fault (58) is 0.26 which is the total time of allowable attack.

Also, with these values of exponential rates, the maximum value for the upper bound of system delay and maximum number of controller failures in this interval respectively are 0.314 and 4.13.

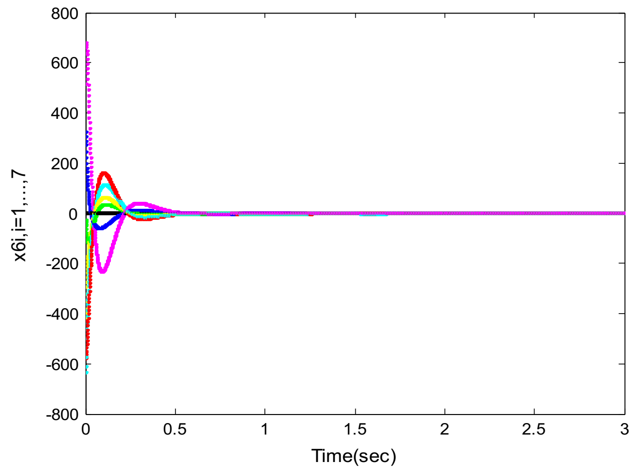

Fig. 7. The behaviors of the $x_{6i}, i=1,...,7$ in the controlled system (57).

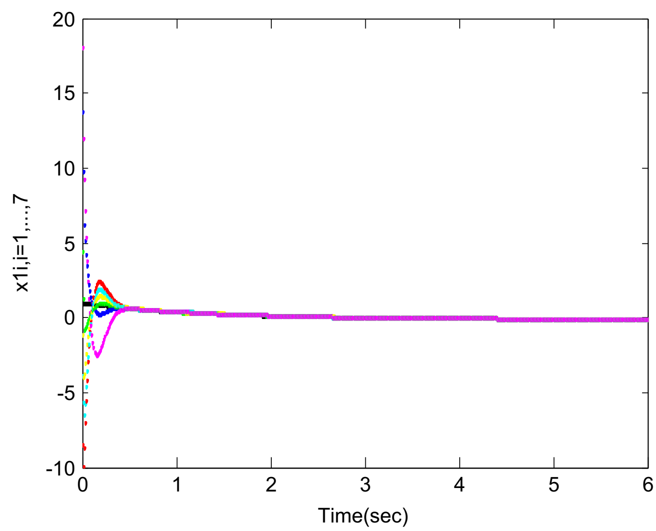

Fig. 8. The behaviors of the $x_{1i}, i=1,...,7$ in the controlled system (57) with fault (58).

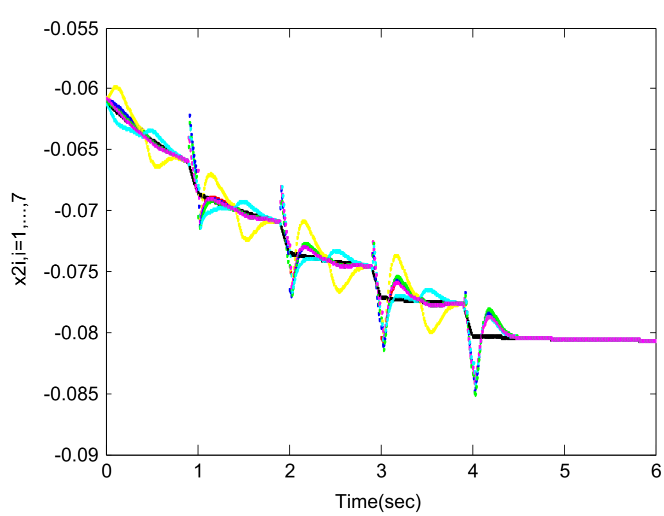

Fig. 9. The behaviors of the $x_{2i}, i=1,...,7$ in the controlled system (57) with fault (58).

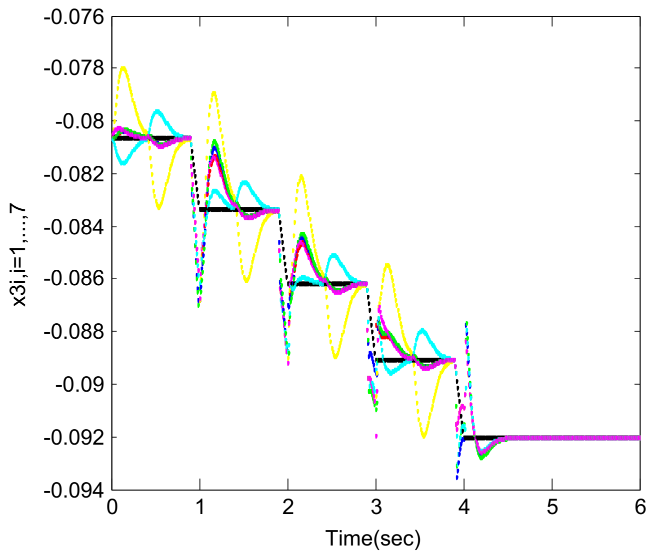

Fig. 10. The behaviors of the $x_{3i}, i=1, \ldots, 7$ in the controlled system (57) with fault (58).

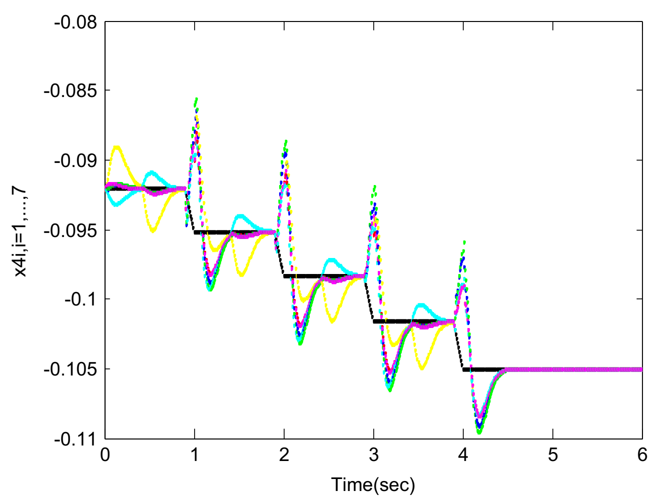

Fig. 11. The behaviors of the $x_{4i}, i=1, \ldots, 7$ in the controlled system (57) with fault (58).

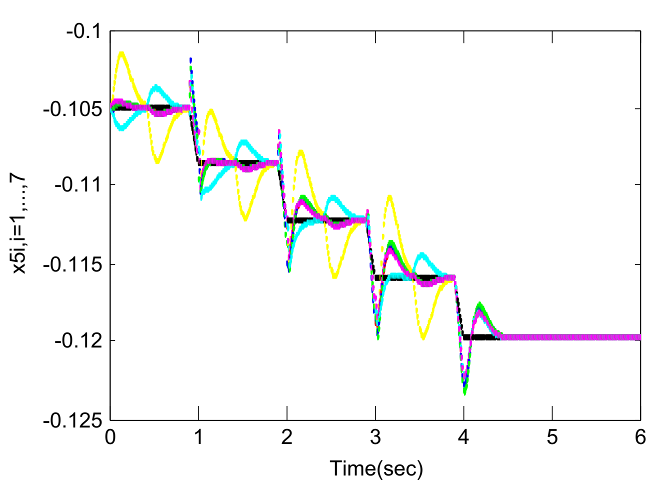

Fig. 12. The behaviors of the $x_{5i}, i=1, \ldots, 7$ in the controlled system (57) with fault (58).

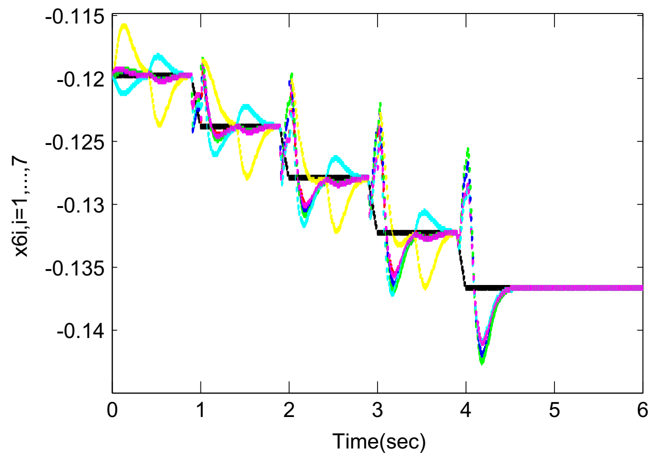

Fig. 13. The behaviors of the $x_{6i}, i=1,...,7$ in the controlled system (57) with fault (58).

It should be emphasized that the appropriate value of ρ, α_1, α_2 for the required calculations is obtained by trial and error method.

6. Conclusions

This paper studied the distributed consensus problem of linear dynamical multi-agent systems with missing control input in some intervals and also delays in the transmission of control input to agents. At first, assuming zero control input in some intervals and delay, the model of system in such conditions was formulated. Then, a distributed adaptive controller based on the relative states of neighboring agents was proposed. By constructing a set of switching Lyapunov–Krasovskii functional, a delay-dependent exponential consensus criterion with explicitly exponential convergence rate was established. Furthermore, the obtained condition was extended to the multi-agent system, when a false signal was injected instead of the nominal control signal. Finally, an illustrative example was solved to show the advantage of the proposed approach. Future possible research directions in this area will be considering fault tolerant of MAS with switching directed topologies and multiple time-varying delays.

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