۱

$$P_c = \frac{1}{N}\sum \rho(x_{ij}, x_{ik}) = \frac{1}{NM(M-1)\sigma_x^2}\sum^{N}\sum^{M}\sum^{M}(x_{ij}-\bar{x}_i)(x_{ik}-\bar{x}_i)$$

اگر حجم خوشه‌ها با هم برابر باشند ⇐

$$\sigma_b^2 = \frac{\sigma_x^2}{M}\left(1+(M-1)P_c\right)$$

$$\sigma_b^2 = \frac{1}{N}\sum^{N}(\bar{x}_{i.}-\bar{x}_{..})^2 = \frac{1}{N}\sum\left(\frac{1}{M}\sum^{M}x_{ij}-\bar{x}_{..}\right)^2$$

$$= \frac{1}{NM^2}\sum^{N}\left(\sum^{M}x_{ij}-M\bar{x}_{..}\right)^2 = \frac{1}{NM^2}\sum\left(\sum^{M}(x_{ij}-\bar{x}_{..})\right)^2$$

$$= \frac{1}{NM^2}\sum^{N}\left(\sum^{M}(x_{ij}-\bar{x}_{..})^2+\sum^{M}\sum^{M}(x_{ij}-\bar{x}_{..})(x_{ik}-\bar{x}_{..})\right)$$

$$= \frac{1}{NM^2}\left(NM\sigma_x^2+P_c MN(M-1)\sigma_x^2\right) = \frac{\sigma_x^2}{M}+\frac{M-1}{M}\sigma_x^2 P_c$$

$$= \frac{\sigma_x^2}{M}\left(1+(M-1)P_c\right) \Rightarrow \begin{cases} \sigma_b^2 \geq 0 \Rightarrow \frac{\sigma_x^2}{M}(1+(M-1)P_c) \geq 0 \Rightarrow P_c \geq -\frac{1}{M-1} \\ \sigma_b^2 \leq \sigma_x^2 \Rightarrow \frac{\sigma_x^2}{M}(1+(M-1)P_c) \leq \sigma_x^2 \Rightarrow P_c \leq 1 \end{cases}$$

$$\Rightarrow \frac{-1}{M-1} \leq P_c \leq 1$$

تحت شرایطی که حجم همه خوشه‌ها با هم برابر باشند

$$\begin{cases} M=1 \rightarrow \sigma_x^2 = \sigma_b^2 \\ P_c=1 \rightarrow \sigma_x^2 = \sigma_b^2 \\ P_c = \frac{-1}{M-1} \rightarrow \sigma_b^2 = 0 \end{cases}$$

۲]

$n = ۳۰ \qquad \rho = \frac{۵}{۶} \qquad C_x = C_y = ۰/۶$

$$\frac{Bias(\bar{y}_r)}{\bar{y}_N} = \left(\frac{1}{n} - \frac{1}{N}\right)\left(C_y^۲ - \rho C_x C_y\right) = \left(\frac{1}{۳۰} - \frac{1}{N}\right)\left((۰/۶)^۲ - \frac{۵}{۶}(۰/۶)(۰/۶)\right)$$

$$\xrightarrow[\frac{1}{N} \approx 0]{N \text{ بزرگ}} \left(\frac{1}{۳۰}\right)\left(۰/۳۶ - \underbrace{\frac{۵}{۶}(۰/۳۶)}_{۰/۳}\right) = \left(\frac{1}{۳۰}\right)(۰/۰۶) = ۰/۰۰۲$$

Scanned with CamScanner

۳

$n = 100 \qquad s^2_{nx} = 1600$

$\widehat{var}(\hat{\bar{x}}_N) = 4$

$(1-f)\dfrac{s^2_{nx}}{n}(1-r^2) = 4 \xrightarrow[\text{چون N بزرگ}]{f \approx 0} (1-0)\dfrac{1600}{100}(1-r^2) = 4$

$(f \to 0)$

$16(1-r^2) = 4 \rightarrow (1-r^2) = \dfrac{1}{4} \qquad \dfrac{3}{4} = r^2 \Rightarrow r = \dfrac{\sqrt{3}}{2}$

$\Rightarrow r = 0/87 = \hat{\rho}$

۴)

$N=100$ $r=4$ (x_{r+jk}, x_{N-r+1})

$k>4$ $k=N/n$ $(x_{4+jk}, x_{100-4+1-jk}) \Rightarrow (x_{4+jk}, x_{97-jk})$

بزرگترین واحد نمونه ۹۷

$\Downarrow$

$y=3i \Rightarrow y_{97} = (3)(97) = 291$

$x_4, x_{4+k}, x_{4+2k}, \ldots, x_{4+(n/2-1)k}$