Genetic Programming to Predict River Pipeline Scour

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Abstract: The process involved in the local scour below pipelines is so complex that makes it difficult to establish a general empirical model to provide an accurate estimation for scour. This technical note describes the use of genetic programming (GP) to estimate the pipeline scour depth. The data sets of laboratory measurements were collected from published literature and used to train the network or evolve the program. The developed network and evolved programs were validated by using the observations that were not involved in the training. The performance of GP was found to be more effective when compared with the results of regression equations and artificial neural networks modeling in predicting the scour depth around pipelines.

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Introduction

Scour is a major cause for the failure of underwater pipelines. Interactions between the pipeline and its erodible bed under strong current and/or wave conditions may cause scour around the pipelines. This process involves the complexities of both the three-dimensional flow pattern and sediment movement. The scour underneath the pipeline may expose a section of the pipe causing it to become unsupported. If the free span of the pipe is long enough, the pipe may experience resonant flow-induced oscillations leading to settlement and potentially structural failure. An accurate estimate of the scour depth is important in the design of submarine pipelines (Chiew 1991). The estimation of the scour characteristics of underwater pipelines continues to be a concern for hydraulic engineers.

A number of empirical formulas have been developed in the past to estimate the equilibrium scour depth below pipelines including Chao and Hennessy (1972), Kjeldsen et al. (1973), Ibrahim and Nalluri (1986), Dutch research group (Bijker and Leeuwestein 1984), Moncada and Aguirre-Pe (1999), and Chiew (1991). However, the main deficiency of these formulas is that the empirical equations do not model actual scour process. Semi-empirical methods combine laboratory and field observations with some physics. Most commonly regression relations are used to predict the pipeline scour; however, the regression analysis can have large uncertainties, which own major drawbacks pertaining idealization of the complex scour process, approximation, and averaging widely varying prototype conditions. Thus, the computed scour depths can be far from the actual ones. Another important issue, apart from the complexity of the scour phenomenon involved, is due to the limitation of the regression analysis. In the regression analysis, whatever the nature of the corresponding problem is, it is tired to model by a predefined equation, either linear or nonlinear. Another major constraint in the application of regression analysis is the assumption of the normality of residuals. A summary of these equations is shown in Table 1.

Predictive approaches such as artificial neural networks (ANNs) (Azmathullah et al. 2005) and adaptive neurofuzzy inference systems (ANFIS) (Azamathulla et al. 2008) have been recently shown to yield effective estimates of the scour around hydraulic structures. ANNs have been reported to provide reasonably good solutions for hydraulic-engineering problems particularly for cases of highly nonlinear and complex relationships among the input-output pairs in the corresponding data (Azamathulla et al. 2010).

The objective of this study is to develop a predictive model for the scour depth using genetic programming (GP). The performance of the proposed GP model is compared with a standard radial basis function (RBF) neural network and conventional regression-based equations. The explicit formulation of the GP model is also presented.

Analysis of Local Scour below Underwater Pipelines

The variables influencing the equilibrium scour depth (d_s) below a pipeline in a steady flow over a "bed of uniform, spherical, and cohesionless sediment as shown in Fig. 1 are: flow condition, sediment characteristics, and pipe geometry." The scour depth can be represented by the following general functional relationship (Moncada and Aguirre-Pe 1999):

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Table 1. Empirical Formulas for Estimate Pipeline Scour Depth

Writer	Equation
Chao and Hennessy (1972)	$q_{bot}=U_0\left(H-\frac{R^2}{2H-R}\right),\quad Ubot=\frac{q_{bot}}{(H-R)}=Uo\left[\frac{2\left(\frac{H}{R}\right)^2-\left(\frac{H}{R}\right)-1}{2\left(\frac{H}{R}\right)^2-3\left(\frac{H}{R}\right)+1}\right]$ for $H\geq R$ and $H-R$=maximum scour depth; R=radius of pipe; q_b=discharge per unit width through the hole; H=distance from bed to pipe center; and U_o=average flow velocity
Kjeldsen et al. (1973)	$ds=0.9722\left(\frac{U_0^2}{2g}\right)^{0.2}D^{0.8}$
Ibrahim and Nalluri (1986)	$\frac{ds}{D}=4.706\left(\frac{U_0}{Uc}\right)^{0.89}\left(\frac{Uo}{gy}\right)^{1.48}+0.06$ clearwater $\frac{ds}{D}=0.084\left(\frac{U_0}{Uc}\right)^{-0.8}\left(\frac{Uo}{\sqrt{gy}}\right)^{-0.16}+1.33$ livebed
Bijker and Leeuwestein (1984)	$ds=0.929\left(\frac{Uo}{2g}\right)^{0.26}D^{0.79}d_{50}^{-0.04}$
Moncada and Aguirre-Pe (1999)	$\frac{ds}{D}=0.9\tanh(1+1.4\mathsf{F})+0.55$ $\frac{ds}{D}=2\mathsf{F}\sec\left(1.7\frac{e}{D}\right)$ D=pipe diameter; e=initial gap between the pipe and undisturbed erodible bed

$$d_s=f(\rho,\rho_s',\nu,Q,Y,g,d_{50},S_f,D) \tag{1}$$

where ρ=fluid density; ρ_s'=buoyant sediment density ; ν=fluid kinematic viscosity; Q=discharge; Y=flow depth; g=gravitational acceleration constant; d_{50}=particle mean diameter; S_f =slope of the energy line; D=the diameter of the pipe; and d_s =equilibrium scour depth.

The nine independent variables in Eq. (1) can be reduced to a set of six nondimensional parameters. The Buckingham pi (or π) theorem applied to Eq. (1), choosing ρ, Q, and D as basic variables, leads to

$$\frac{d_s}{D}=\Psi\left(\tau_*,\frac{Y}{D},\frac{D}{d_{50}},\mathsf{R},S_f,F\right) \tag{2}$$

where τ_*=dimensionless Shields parameter related to sediment transport; D/d_{50}=dimensionless soil characteristics; $\mathsf{R}=VD/\nu$ =Reynolds number; S_f=slope of the energy line; and $\mathsf{F}=V/\sqrt{gY}$ =Froude number. The influence of the Reynolds number is considered negligible under a fully turbulent flow over a rough bed (Lim and Chiew 2001; Melville 1992). The experimental data were collected from several references such as Moncada and Aguirre-Pe (1999) and Dey and Singh (2008). The whole data set consists of 215 data sets.

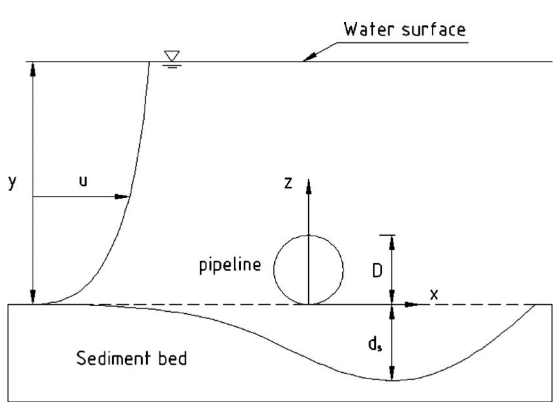

Fig. 1. Local scour below pipeline in river crossing (Dey and Singh 2008)

During the past two decades, researchers have noticed that the use of soft computing techniques as an alternative to conventional statistical methods based on controlled laboratory or field data yielded significantly better results. The ANN and GP are the most widely used branches of soft computing in hydraulic engineering. Within the larger field of hydraulics, several researchers have dealt with the scour around and downstream of hydraulic structures using ANNs (Azmathullah et al. 2005; Azamathulla et al. 2010, 2008). Gene-expression programming (GEP), which is an extension of GP, recently has attracted the attention of researchers in the prediction of hydraulic characteristics. This study presents the ANN and GP as an alternative tool in the prediction of scour below the pipeline.

Development of Neural Network Model

ANNs provide a random mapping between an input and an output vector, typically consisting of three layers of neurons, namely, input, hidden, and output, with each neuron acting as an independent computational element. Neural networks derive their strengths from the high degree of freedom associated with their architecture. Prior to application, the network is trained to observe data sets. This feeds the network with input and output pairs and determines the values of connection weights, bias, or centers (Fig. 2 as example). The training may require many epochs (presenta-

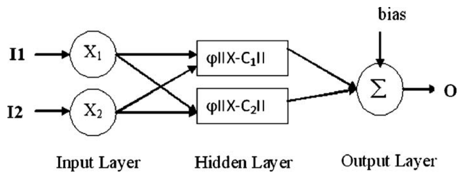

Fig. 2. RBF neural network architecture

tion of complete data sets once to the network), being carried out until the training sum of squares error reaches a specified error goal. The concepts involved behind these training schemes are outlined in the American Society of Civil Engineers (ASCE) Task Committee (2000). A neural network toolbox contained within the MATLAB package was used in this study. The usual feedforward type of network was trained using the RBF. Out of the total of 215 input-output pairs, about 75% (161 sets), selected randomly, were used for training, whereas the remaining 25% (54 sets) were employed for testing. As dictated by the use of a Gaussian function, all patterns were normalized within the range of (0.0, 1.0) before their use. The RBF network (five inputs, 36 hidden neurons, and one output) was trained by using various values of spread (α) between 0 and 1. A spread constant α for the radial basis layer, and returns a network with weights and biases such that the outputs are exactly for given targets. The value of 0.01 was selected as it yielded the best performance for the training data.

Development of GP Model

GP, a branch of the genetic algorithm (GA) (Holland 1975), is a method for learning the most "fit" computer programs by means of artificial evolution (Johari et al. 2006). GP initializes a population consisting of the random members known as chromosomes (individual), and the fitness of each chromosome is evaluated with respect to a target value. The principle of Darwinian natural selection is used to select and reproduce "fitter" programs. GP creates equal or unequal length computer programs that consist of variables (terminal) and several mathematical operators (function) sets as the solution. The function set of the system can be composed of arithmetic operations $(+,-,/,\times)$ and function calls such as $(e^x, x, \sin, \cos, \tan, \log, \mathrm{sqrt}, \ln, power)$. Each function implicitly includes an assignment to a variable, which facilitates the use of multiple program outputs in GP, whereas in tree-based GP those side effects need to be incorporated explicitly (Brameier and Banzhaf 2001).

The GP used in this study utilizes a two-point string crossover. A segment of random position and random length is selected in both parents and exchanged between them. If one of the resulting children would exceed the maximum length, the crossover is abandoned and restarted by exchanging equalized segments (Brameier and Banzhaf 2001). An operand or an operator of an instruction is changed by mutation into another symbol over the same set.

The fitness of a GP individual may be computed by using the equation

$$f = \sum_{j=1}^{N} (|X_j - Y_j|) \tag{3}$$

where X_j=value returned by a chromosome for the fitness case j; and Y_j=expected value for the fitness case j.

In GP, the maximum size of the program is usually restricted to avoid overgrowing programs without bounds (Brameier and Banzhaf 2001). This configuration was tested for the proposed GP model and was found sufficient. The best individual (program) of a trained GP can be converted into a functional representation by successive replacements of variables starting with the last effective instruction (Oltean and Groşan 2003).

To date, the application of GP in hydraulic engineering has been limited. Davidson et al. (1999) and Babovic and Keijzer (2000) determined empirical relationships for the friction in turbulent pipe flow and the additional resistance to flow induced by flexible vegetation, respectively. Keijzer and Babovic (2002) derived empirical equations using real-world hydraulic data. Giustolisi (2004) determined Chezy resistance coefficients in corrugated-metal pipes. Kizhisseri et al. (2005) explored a better correlation between the temporal pattern of flow field and sediment transport by using numerical model results and field data; and Azamathulla et al. (2010) predicted local scour at bridge piers.

The GP model was developed using the same input variables as with an ANN-RBF model. Five of 10 parameters in Eq. (1), namely, the fluid density, the buoyant sediment density, fluid dynamic viscosity, gravitational acceleration, and the slope of the energy line are constant in all experiments. Therefore, the first combination involves just four of the 10 parameters in Eq. (1) as the input pattern and the equilibrium scour depth (d_s) as the output pattern. The second combination includes the six nondimensional parameters of Eq. (2) and the normalized equilibrium scour depth (d_s/D) as the input and output patterns, respectively. Both of these combinations of inputs have been used for the GP and ANN models.

In this study, four basic arithmetic operators $(+,-,\times,/)$ and some basic mathematical functions ($\sqrt{}$, x^2, power) were used. A large number of generations (5,000) were tested. First, the maximum size of each program was specified as 256, starting with 64 instructions for the initial program.

The simplified analytic form of the proposed GP model may be expressed as

$$d_s/D = (d_{50}/D)^{-0.5}\left[\left(\left\{\left[-1.36\frac{(1-\mathsf{T}^2)^2(Y/D)^{0.5}}{(d_{50}/D)}+\mathsf{T}^2\right]^2+\mathsf{F}\right\}^2 - 1\right)^2 - \tau 0 - 1\right]^{0.5} \tag{4}$$

where

$$T = 2\left[\frac{\frac{(\mathsf{F}-0.314)}{Y/D}+\mathsf{F}-\mathsf{R}-0.739}{d_{50}/D}+(\mathsf{F}-0.224)\right]^2$$

Training and Testing Results of GP Modeling

The performance of GP in training and testing sets is validated in terms of the common statistical measures R^2 (coefficient of determination), root mean square error (RMSE), mean average error (MAE), and δ (average absolute deviation).

Table 2 shows the range of variation of collected data for this study and its parameters. The functional set and operational parameters used in GP modeling during this study are listed in Table 4.

Table 2. Data Variation

Parameters	Unit	Data range	Mean	Standard deviation
(a) Range of different input-output parameters used for the estimation of scour depth				
Flow discharge (Q)	cm^3/s	7–94.42	35.11	21.74
Flow depth (Y)	cm	3.8–28	13.43	6.21
Particle mean diameter (d_{50})	cm	0.234–0.7	0.437	0.144
Diameter of the pipe (D)	cm	0.48–7.6	1.92	1.61
Equilibrium scour depth (d_s)	cm	0.02–11.3	4.75	2.39
(b) Range of different nondimensional input-output parameters used for the estimation of scour depth				
Dimensionless Shields parameter (τ_*)		0.038–0.70	0.23	0.17
Normalized flow depth (Y/D)		1.06–7	3.14	1.2
Pipeline diameter cross section of sediment size (D/d_{50})		3.28–145.8	38.17	31.41
Froude number (F)		0.2–0.83	0.46	0.15
Reynolds number R is normally used		700–9,450	3,250	2,174
Nondimensional equilibrium scour depth ($-$)		0.008–1.66	1.04	0.32

The performance of all models was compared using four error measures

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(o_i - t_i)^2}{\sum_{i=1}^{N}(o_i - \bar{o}_i)^2} \tag{5}$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(o_i - t_i)^2}{N}} \tag{6}$$

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}|o_i - t_i| \tag{7}$$

$$\delta = \frac{\sum|(o_i - t_i)|}{\sum o_i} \times 100 \tag{8}$$

Table 3. Parameters of the Optimized GP Model

Parameter	Description of parameter	Setting of parameter
p_1	Function set	$+,-,\times,/,\sqrt{}$, power
p_2	Population size	250
p_3	Mutation frequency (%)	96
p_4	Crossover frequency (%)	50
p_5	Number of replication	10
p_6	Block mutation rate (%)	30
p_7	Instruction mutation rate (%)	30
p_8	Instruction data mutation rate (%)	40
p_9	Homologous crossover (%)	95
p_{10}	Program size	Initial 64; maximum 256

Table 4. Sensitivity Analysis for Independent Parameters for the Testing Set

Model	RMSE	MAE	R^2
$\frac{d_s}{D} = \Psi\left(\tau_*, \frac{Y}{D}, \frac{D}{d_{50}}, R_p, \mathsf{F}\right)$	0.046	0.32	0.85
$\frac{d_s}{D} = \Psi\left(\frac{Y}{D}, \frac{D}{d_{50}}, R_p, \mathsf{F}\right)$	0.065	0.45	0.82
$\frac{d_s}{D} = \Psi\left(\tau_*, \frac{D}{d_{50}}, R_p, \mathsf{F}\right)$	0.075	0.53	0.89
$\frac{d_s}{D} = \Psi\left(\tau_*, \frac{Y}{D}, R_p, \mathsf{F}\right)$	0.098	0.76	0.71
$\frac{d_s}{D} = \Psi\left(\tau_*, \frac{Y}{D}, \frac{D}{d_{50}}, \mathsf{F}\right)$	0.134	0.87	0.85
$\frac{d_s}{D} = \Psi\left(\tau_*, \frac{Y}{D}, \frac{D}{d_{50}}, R_p\right)$	0.256	0.987	0.656

where t_i denotes the target values of the equilibrium scour depth (cm); while o_i and $\bar{o}_i$ denote the observed and averaged observed values of the equilibrium scour depth (cm), respectively; and N =number of data points. First, an attempt was made to assess the significance or influence of each input parameter on estimated ds/D values. Table 3 compares the GP models, with one of the independent parameters removed in each case and deleting any independent parameter from the input set yielded larger RMSE and lower R^2 values. These five independent parameters have influence on ds/D and so the functional relationship given in Eq. (2) is used for the GP modeling in this study. The GP approach resulted in a highly nonlinear relationship between ds/D and the input parameters with high accuracy and relatively low error. The testing performance of the proposed GP model revealed a high generalization capacity with R^2=0.89, RMSE=0.046, MAE =0.32%, and δ=9.9.

Results and Discussion

In this study, different combinations of input data (nondimensional data set) were explored to assess their influence on the scour depth modeling (Table 3). The GP model was developed and tested for predicting the pipeline scour depth. The dimensional parameter combinations included the flow discharge; flow depth; particle mean diameter; diameter of the pipe; and the equilibrium scour depth. A dimensional analysis was used to determine the parameter for underwater pipeline scour. The nondimensional parameters combination include the dimensionless Shields parameter related to sediment transport; pipeline di-

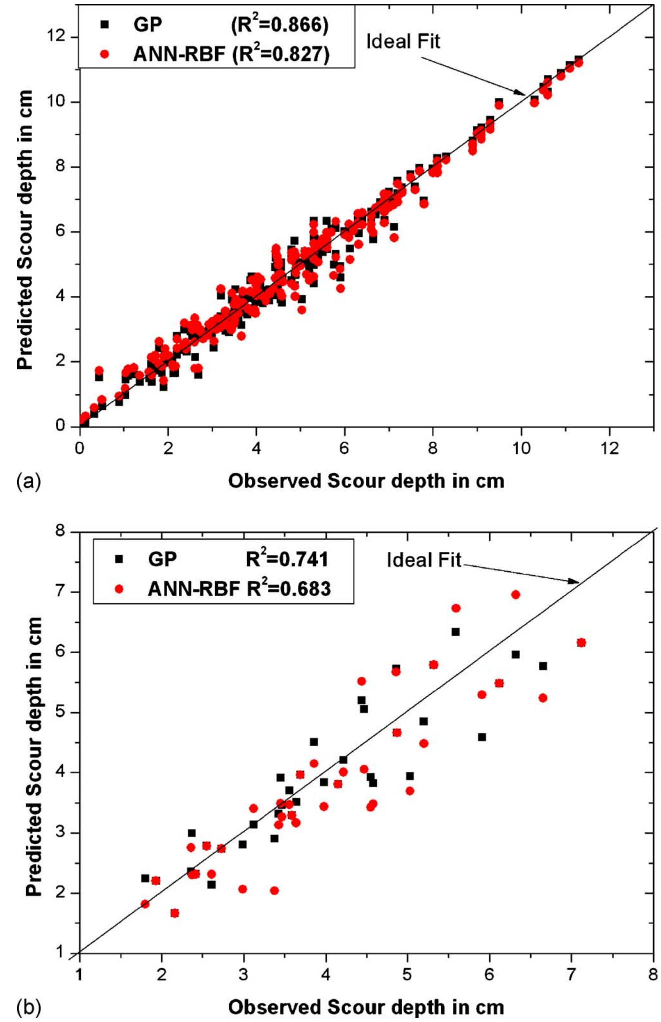

Fig. 3. Observed versus predicted scour depth (training and testing)

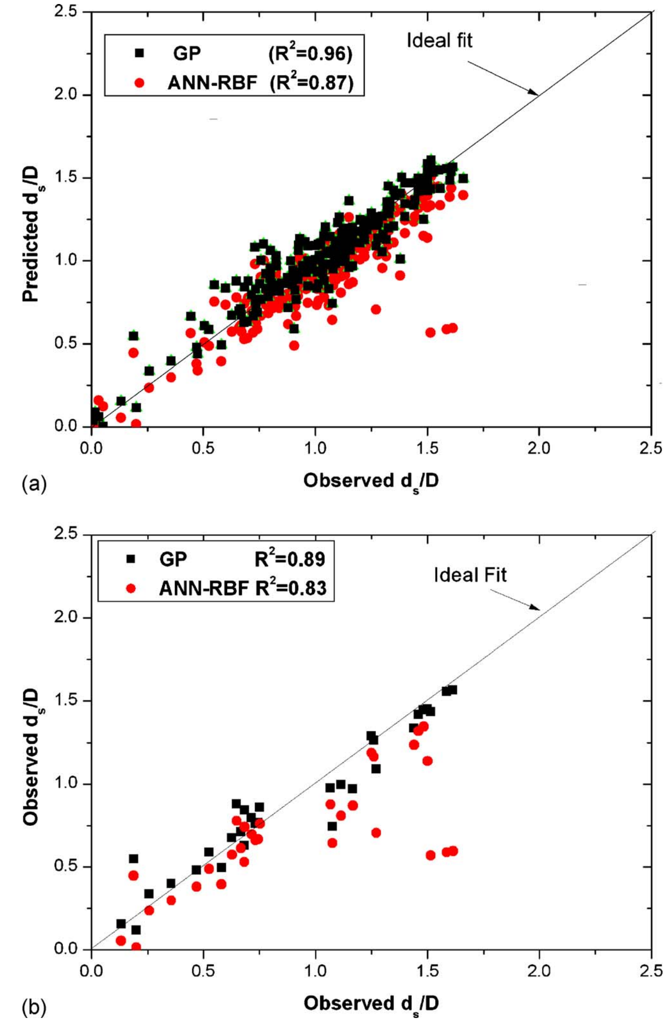

Fig. 4. Observed versus predicted scour depth (training and testing)

ameter cross section of grain size (d_{50}); and the Froude number. Each parameter (except energy slope) in Eqs. (1) and (2) was considered in turn in the GP for the sensitivity analysis. The results show that, of the parameters in Eq. (1), the mean particle size (d_{50}) has the most significant effect on the scour depth and the flow discharge has the least effect on it.

Similarly, for the nondimensional parameter in Eq. (2) sensitivity analysis shows that the dimensionless Shields parameter (τ_*) and Y/D have, respectively, the most and the least effect on the normalized scour depth. To assess the performance of the GP model, the observed equilibrium scour depth values were plotted against the predicted ones. Figs. 3 and 4 illustrate the results with the performance indices between the predicted and observed data for the training and validating (testing) data sets for the dimensional parameters. Fig. 4 shows the same nondimensional respectively for both models. As can be seen from Table 5, the first combination (original data) has a better ability to predict the scour depth (R^2=0.741). The result of the second combination data shows a high coefficient of determination [R^2=0.89, also RMSE (=0.046)] in the second combination is better than the first combination (RMSE=0.0957) in both the training and validation periods but this variation is low compare with the R^2 variation.

Table 5. Comparison of Models for Dimensional and Nondimensional Sets Performance of the GP and ANN-RBF

	R^2		RMSE		MAE		δ	
	Training	Validation	Training	Validation	Training	Validation	Training	Validation
Models for dimensional								
GP	0.866	0.741	0.0895	0.0957	1.279	1.426	5.78	10.45
ANN-RBF	0.827	0.683	0.0978	0.0998	1.933	2.71	11.49	15.67
Models for nondimensional								
GP	0.96	0.89	0.029	0.046	0.279	0.320	3.7	9.9
ANN-RBF	0.87	0.73	0.008	0.073	0.083	0.071	11.45	15.67

Conclusion

The application of the relatively new soft computing approach of GP to predict the local pipeline scour depth was described. A GP and an ANN-RBF model were developed to predict the values of the relative scour depth from the laboratory measurements. A new approach was presented to estimate the equilibrium depth scour below underwater pipelines in a river crossing from optimum data sets with the GP and ANN modeling techniques. The application of the GP in this study is another important contribution to scour-depth estimation methodologies for pipes. The present study indicates that employing the original data set yielded a network that can predict measured pipeline depth scour in rivers more accurately than standard regression analysis. The overall performance of the GP model is superior to the ANN model.

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Notation

The following symbols are used in this technical note:

D = diameter of pipe;
d_s = equilibrium scour depth;
d_{50} = particle mean diameter;
F = Froude number;
g = gravitational acceleration;
MAE = mean average error;
Q = discharge;
q_{bot} = discharge per unit width through hole;
R = Reynolds number;
R^2 = coefficient of determination;
RMSE = root-mean-square error;
S_f = slope of energy line;
V = flow velocity;
Y = flow depth;
α = spread;
δ = average absolute deviation;
ν = fluid kinematic viscosity;
ρ = fluid density;
ρ_s' = buoyant sediment density; and
τ_* = dimensionless Shields parameter.

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