ANFIS-Based Approach for Predicting the Scour Depth at Culvert Outlets

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Abstract: The processes involved in the local scour at culverts are so complex and that makes it difficult to establish a general empirical model to provide accurate estimation for scour. This paper describes the use of adaptive neurofuzzy inference system (ANFIS) to estimate the scour depth at culvert outlets. The data sets of laboratory measurements were compiled from published literature and used to train the ANFIS network. The developed network was validated by using the observations that were not involved in training. The performance of ANFIS was found to be more effective (R^2=0.94) when compared with the results of regression equations and artificial neural networks modeling in predicting the scour depth at culvert outlets (R^2=0.78). Further work is required to collect field data of scour at culvert outlets to train the genetic programming approach and validate its usefulness.

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Introduction

An essential feature in the hydraulic design of drainage-crossing hydraulic structures such as culverts or storm drains is determining the design capacity of flow capacity (Lim 1995). Accurate prediction of the dimensions of scour downstream from hydraulic structures is required to ensure foundations are properly designed and prevent damage to the structure as a result of undermining (Liriano and Day 2001). The estimation of scour characteristics at culvert outlets (Fig. 1) continues to be a concern for hydraulic engineers.

A number of empirical formulas have been developed in the past to estimate equilibrium scour depth at culvert outlets, including Opie (1967), Rajaratnam and Berry (1977), Rajaratnam (1981), Ruff et al. (1982), Rajaratnam and MacDougall (1983), Blaisdell and Anderson (1988), Abida and Townsend (1991), Lim (1995), and Chiew and Lim (1996). These traditional scour prediction equations (Table 1), although offering the engineer some guidance on the likely magnitude of maximum scour depth, are applicable only to a limited range of field conditions. A model for the prediction of scour downstream from culverts that is generally applicable to all circumstances is not currently available. However, the main deficiency of these formulas is that the empirical equations do not model actual scour processes. Most commonly, regression relations are used to predict culvert outlet scour; however, regression analysis can have large uncertainties, which include major drawbacks pertaining to idealization of complex scour process, approximation and averaging widely varying prototype conditions. Thus, the estimated scour depths using regression equations can have large uncertainties, which can contribute to costly culvert failures.

Predictive approaches such as artificial neural networks (ANN) (Azmathullah et al. 2005) and adaptive neurofuzzy inference systems (ANFIS) (Azamathulla et al. 2008) have been recently shown to yield effective estimates of scour around hydraulic structures. ANNs have been reported to provide reasonably good solutions for hydraulic-engineering problems, particularly for cases of highly nonlinear and complex relationship among the input-output pairs in corresponding data (Guven and Gunal 2008a; Azamathulla et al. 2010; Azamathulla and Ghani 2010).

The objective of this study is to develop an improved predictive model for estimating scour depth using ANFIS. The performance of the proposed ANFIS model is compared with a standard radial basis function (RBF) neural network and conventional regression-based equations (Lim 1995; Chiew and Lim 1996; Abt et al. 1984).

Analysis of Local Scour at Culvert Outlets

The variables influencing the equilibrium scour depth (d_s) at culvert outlets are listed as below (Liriano and Day 2001)

$$d_s = f(\rho, \mu_0, u_0, d_0, H, W, W_0, g, \rho_s', d_{50}, \sigma_g, K_s) \tag{1}$$

where d_s=maximum depth of scour; ρ=density of water; μ_0 =dynamic viscosity of water; u_0=mean velocity at the outlet; d_0 =pipe diameter for circular outlets and the outlet height for noncircular outlets; H=depth of water in the downstream receiving channel (tailwater depth); W=width of the receiving channel; W_0=width of the outlet; g=acceleration due to gravity; ρ_s =density of the sediment bed material; d_{50}=median sediment

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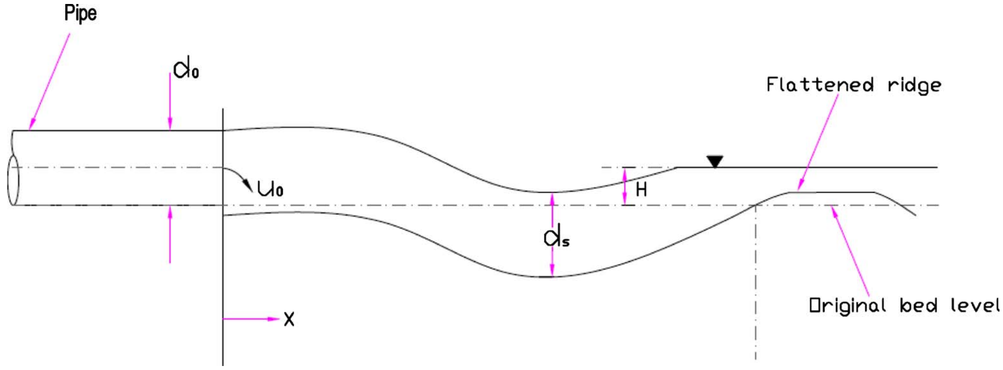

Fig. 1. Typical center-line bed profile below circular pipe outlet at equilibrium scour condition (Lim 1995, with permission from the Institution of Civil Engineers and S.Y. Lim)

size; K_s=shape factor of a culvert; and σ_g=geometric standard deviation of the sediment bed material and describes the gradation of sediments downstream from the culvert. Assuming that the viscous effect is not important and that the bed material consists of sand and gravel with constant ρ_s, a dimensional analysis Eq. (1) can be reduced to a set of five nondimensional parameters, it gives

$$\frac{d_s}{d_0}=f\left(\mathsf{F}_0,\frac{H}{d_0},\frac{W}{d_0},\frac{W_0}{d_0},\frac{d_{50}}{d_0},\sigma_g,K_s\right) \tag{2}$$

where $\mathsf{F}_0=u_0/[(S-1)gd50]^{0.5}$=densimetric Froude number and $S=\rho_s/\rho$=specific gravity of the sediment. Experimental data were compiled from seven papers such as Bohan (1970), Ruff et al. (1982), Ali and Lim (1986), Abida and Townsend (1991), Lim (1995), Ade and Rajaratnam (1998), and Aderibigbe and Rajaratnam (1998). The compiled data set consists of 202 data sets.

During last two decades, researchers have noticed that the use of soft computing techniques [ANN, ANFIS, genetic programming (GP), etc.] as alternative to conventional statistical methods based on controlled laboratory or field data yielded significantly better results for spillway and bridge pier scour. ANN and ANFIS are the most widely used branches of soft computing in hydraulic engineering. Within the larger field of hydraulics, several researchers have dealt with the scour around and downstream of hydraulic structures using ANN (Azmathullah et al. 2005, 2008; Guven and Gunal 2008a,b). ANFIS, which is an extension of ANN with hybrid networks (neurofuzzy), recently has attracted the attention of researchers in prediction of hydraulic characteristics. This study presents ANN and ANFIS as alternative tool in the prediction of scour depth at culvert.

Development of the Neural Network Model

ANN provides a random mapping between an input and an output vector, and typically consists of three layers of neurons namely, input, hidden and output, with each neuron acting as an independent computational element. Neural networks derive their strengths from the high degree-of-freedom associated with their architecture. Prior to application, the network is trained (calibrated) to observed data sets. This feeds the network with input and output pairs and determines the values of connection weights in the hidden layer, bias or centers (Fig. 2 as example).

The output y of a RBF network corresponding to input x is computed by the equation

$$y=f(x)=\sum_{i=1}^{n} w_i R_i(x)+\theta \tag{3}$$

where w_i=connection weight between the hidden neuron and output neuron; θ=bias; and $R_i(x)$=RBFs given by (Fig. 2)

$$R_i(x)=\varphi\|x-c_i\| \tag{4}$$

having a maximum value at the origin that decays rapidly as its argument tends to infinity. It approaches zero as the Euclidean distance increases between an input vector and the center increases. The general class of RBFs is Gaussian

$$R_i=-\exp\left(-\sum_{i=1}^{n}\frac{\|x_i-c_i\|^2}{2\sigma_{ij}^2}\right) \tag{5}$$

where $c_i^T=[c_{i1},c_{i2},c_{i3},\ldots,c_{in}]$=center of the receptive field and α_{ij}=width of the Gaussian function which indicates the selectivity of the neuron. The major task of RBF network design is to determine center c. The simplest and easiest way may be to choose the centers randomly from the training set. The second approach is to use the k-means technique of clustering input training set into groups and choose the center of each group as the center. Also, c can be treated as a network parameter along with w_i and adjusted through error-correction training. After the center is determined, the connection weights w_i between the hidden layer and

Table 1. Empirical Formulas to Estimate the Scour Depth at Culvert Outlets

Writer	Equation
Lim (1995)	$d_{se}/d_o=0.45\ \mathsf{F}_0$
Chiew and Lim (1996)	$d_{se}/d_o=0.21\ \mathsf{F}_0$
Abt et al. (1984)	$d_{se}/d_0=-3.67(\mathsf{F}_0)^{0.57}(d_{50})^{0.4}(\sigma_g)^{-0.4}$

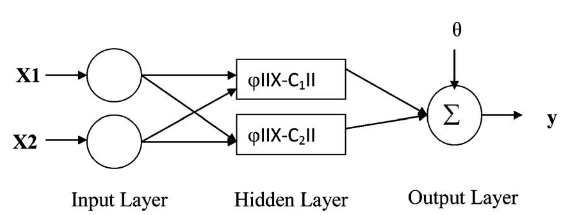

Fig. 2. RBF neural network architecture

Table 2. Range of Data Used

Variable	Range of data
Outlet shape	Circular and box
Culvert shape	Rectangular Circular Square
Outlet diameter d_0 (m)	0.0254–0.146
Sediment size d_{50}/d_0	0.016–0.28
Tailwater depth H/d_0	0.5–25
Exit velocity u_0 (m/s)	0.747–11.176
F_0	1.04–88.61
W/d_0	1.0–86.0

output layer can be determined simply through ordinary back-propagation training.

A neural network toolbox contained within the MATLAB (2009) package was used in this study. The usual feed-forward type of network was trained using a RBF. Out of the total of 202 input-output pairs, about 75% of data sets (151 sets) selected randomly and were used for training, whereas the remaining 25% of data sets (51 sets) were employed for testing (model validation). Table 2 shows the variables and their ranges of the complied data used in this study. As dictated by the use of a Gaussian function, all patterns were normalized within the range of 0.0, 1.0 before their use. The RBF network [7 inputs, 32 hidden neurons, and 1 output as in Eq. (2)] was trained using various values of spread (α_{ij}) between 0 and 1. A spread constant α for the radial basis layer, and returns a network with weights and biases such that the outputs are exactly for given targets. The value of 0.01 was selected as it yielded the best performance for the training data.

ANFIS Networks

The ANFIS, first introduced by Jang (1993), is a universal approximator and, as such, is capable of approximating any real continuous function on a compact set to any degree of accuracy (Jang 1993).Thus, in parameter estimation, where the given data sets are such that the system associates measurable system variables with an internal system parameter, a functional mapping is constructed by ANFIS that approximates the process of estimation of the internal system parameter.

The ANFIS is functionally equivalent to fuzzy inference systems (Jang 1993). Below, the hybrid learning algorithm (Jang 1993) which combines gradient descent and the least-squares method, is introduced, and the issue of how the equivalent fuzzy inference system can be rapidly calibrated and adapted with this algorithm is discussed.

Most of the previous works that address ANN applications to water resources have included the feed forward type of the architecture, where there are no backward connections, which are trained using the error back-propagation scheme or the feed forward back-propagation (FFBP) network configuration. Drawbacks of ANN include that it needs substantial training time and the difficulties in detecting hidden neurons in hidden layer for better predictions. Therefore, the present study applies a new soft computing technique ANFIS.

The input in ANFIS is first converted into fuzzy membership functions, which are combined together. After following an averaging process to obtain the output membership functions, the desired output is finally achieved.

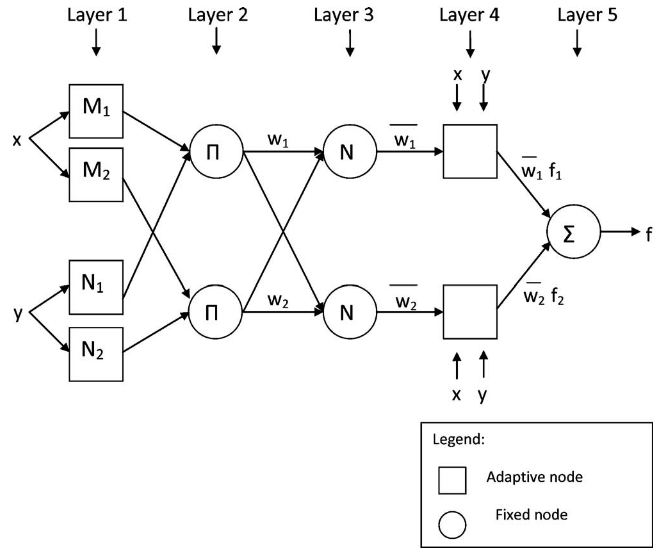

Fig. 3. ANFIS network architecture

Development of ANFIS Model

The ANFIS network (Fig. 3) works as follows: Let x and y be the two typical input values fed at the two input nodes, which then transforms those values to the membership functions (say bell-shaped) and give the output as follows: [Note in general, w=output from a node; μ=membership function; and M_i and N_i=fuzzy sets associated with nodes x, y in Eq. (6).]

$$\mu_{Mi}(x) = \frac{1}{1 + |(x - c_1)/a_1|^{2b_1}} \tag{6}$$

where a_1, b_1, and c_1=changeable basis parameters. Similar computations are carried out for the input of y to obtain $\mu_{Ni}(y)$. The membership functions are then multiplied in the second layer, e.g.

$$w_i = \mu_{Mi}(x) \,.\, \mu_{Ni}(y) \quad (i = 1,2) \tag{7}$$

Such products or firing strengths are then averaged

$$\overline{w_i} = w_i/\sum w_i \quad (i = 1,2) \tag{8}$$

Nodes of the fourth layer use the above ratio as a weighting factor. Furthermore, using fuzzy if-then rules produces the following output: (An example of an if-then rule is: If x is M_1 and y is N_1, then $f_1 = p_1x + q_1y + r_1$)

$$\overline{w_i}f_i = \overline{w_i}(p_ix + q_iy + r_i) \tag{9}$$

where p, q, and r=changeable consequent parameters. The final network output f is produced by the node of the fifth layer as a summation of all incoming signals, which is exemplified in the Eq. (9). The parameters like p, q, and r employed in Eq. (9), for each rule of the ANFIS models, are given in Table 3. The corresponding rules of the developed ANFIS model are listed in Table 4.

A two-step process is used for faster calibrating and to adjust the network parameters to the above ANFIS network. In the first step, the premise parameters are kept fixed, and the information is propagated forward in the network to Layer 4. In Layer 4, a

Table 3. Parameters of the ANFIS Models

Rule parameters	ANFIS
Rule 1	−1.236
	−0.03245
	0.3489
	−0.04467
	158
	14.95×10^{-6}
	−25.82
Rule 2	2570
	0.0635
	−2550
	98.43
	$-4.034\times10^{+4}$
	-2.583×10^{-6}
	$2.058\times10^{+4}$

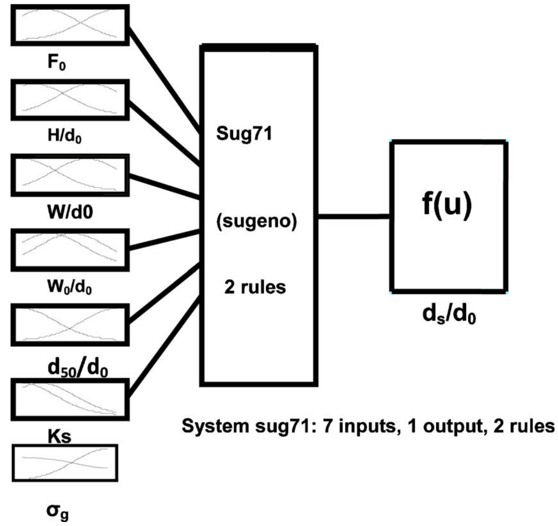

Fig. 4. ANFIS—model (inputs and output)

least-squares estimator identifies the important parameters. In the second step, the backward pass, the chosen parameters are held fixed while the error is propagated. The basis parameters are then modified using gradient descent. Apart from the calibration patterns, the only user-specified information required is the number of membership functions for each input. The description of the learning algorithm is given in Jang and Sun (1995).

The ANFIS model was developed using the same input variables as with an ANN-RBF model as in Eq. (2). The gravitational acceleration and the slope of energy line are constant in all experiments. The seven nondimensional (grouped) parameters of Eq. (2), and normalized equilibrium scour depth (d_s/d_0) are the input and output patterns, respectively. It is obvious that nondimensional (grouped) parameters should be used all the time in analyzing results in the engineering community (Azmathullah et al. 2005). The following scenarios are considered in building the ANFIS model (Fig. 4) with the inputs and output shown in the network. A computer program (MATLAB code) was developed to perform the analysis, and can be obtained from the corresponding writer.

Training and Testing Results of ANFIS Modeling

The performance of ANFIS model in training and testing sets is validated in terms of the common statistical measures; R^2 (coefficient of determination), RMSE, MAE, and δ (average absolute deviation). The functional set and operational parameters used in the ANFIS modeling in this study is listed in Table 3.

A quantitative comparison is shown in Table 5 referred in terms of the four error measures. The performance of all models was compared using above four error measures and expressions for these measures are given below

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(o_i - t_i)^2}{\sum_{i=1}^{N}(o_i - \bar{o}_i)^2} \tag{10}$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(o_i - t_i)^2}{N}} \tag{11}$$

$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N}|o_i - t_i| \tag{12}$$

$$\text{and} \quad \delta = \frac{\sum|(o_i - t_i)|}{\sum o_i} \times 100 \tag{13}$$

where t_i denotes the target values of equilibrium scour depth (cm), while o_i and $\bar{o}_i$ denote the observed and averaged observed values of equilibrium scour depth (cm), respectively, and N = number of data points namely, (1) coefficient of determination, R^2, which presents the degree of association between predicted and true values; (2) MAE (+ or −), which is a parameter commonly understood in engineering applications and which considers algebraic difference between predicted and true values; (3) RMSE, which is preferred in many iterative prediction and opti-

Table 4. Rules for ANFIS Models (Refer to Fig. 4)

Rules	ANFIS
1	1. If (F_0 is F_0mf1) and (H/d_0 is H/d_0mf1) and (W/d_0 is W/d_0mf1) and (d_{50}/d_0 is d_{50}/d_0mf1) and (K_s is K_smf1) and (σg is $\sigma gmf1$) then (d_s/d_0 is d_s/d_0mf1) (1)
2	2. If (F_0 is F_0mf1) and (H/d_0 is H/d_0mf1) and (W_0/d_0 is W_0/d_0mf1) and (d_{50}/d_0 is d_{50}/d_0mf1) and (K_s is K_smf1) and (σg is $\sigma gmf1$) then (d_s/d_0 is d_s/d_0mf1) (2)

Table 5. Comparison of the ANFIS and ANN-RBF Models

Error	Training		Validation	
measure	ANN-RBF	ANFIS	ANN-RBF	ANFIS
R^2	0.842	0.978	0.783	0.941
RMSE	0.0046	0.0795	0.0978	0.0046
MAE	1.945	1.516	2.87	1.426
δ	11.21	5.23	15.34	9.90

Table 6. Sensitivity Analysis for Independent Parameters for the Testing Set

Model	RMSE	MAE	R^2
$d_s/d_0=f(\mathsf{F}_0, H/d_0, W/d_0, W_0/d_0, d_{50}/d_0, \sigma_g, K_s)$	0.046	0.32	0.941
$d_s/d_0=f(\mathsf{F}_0, H/d_0, W/d_0, d_{50}/d_0, \sigma_g, K_s)$	0.065	0.45	0.82
$d_s/d_0=f(\mathsf{F}_0, H/d_0, W_0/d_0, d_{50}/d_0 K_s, \sigma_g)$	0.075	0.53	0.84
$d_s/d_0=f(\mathsf{F}_0, H/d_0, W/d_0 d_{50}/d_0, \sigma_g)$	0.058	0.76	0.74
$d_s/d_0=f(\mathsf{F}_0, W/d_0, d_{50}/d_0, K_s, \sigma_g)$	0.134	0.87	0.82

mization schemes; and (4) the average absolute deviation, δ, which does not even out positive or negative errors as in MAE. First, an attempt was made to assess the significance or influence of each input parameter on estimated d_s/d_0 values. Table 6 provides a summary of the ANFIS models, with one of the independent parameters removed in each case, and deleting any independent parameter from the input set yielded larger RMSE and lower R^2 values. These five independent parameters have influence on d_s/d_0 and so the functional relationship given in Eq. (2) is used for the ANFIS modeling in this study. The ANFIS approach resulted in highly nonlinear relationship between d_s/d_0 and the input parameters with high accuracy and relatively low error. The testing performance of the proposed ANFIS model revealed a good predictive capacity to yield acceptable error measures with $R^2=0.941$, RMSE=0.046, MAE=1.426%, and $\delta=9.9$.

Results and Discussion

In this study, grouped variables (nondimensional data set) of input data were explored to assess their influence on the scour-depth modeling (Table 6). The ANFIS model was developed and tested for predicting culvert scour depth. Dimensional analysis was used to determine parameter for scour at culvert outlet. A nondimensional parameter in the Eq. (2) sensitivity analysis shows that dimensionless shape factor parameter (K_s) and d_{50}/d_0 have, respectively, the most and the least effect on normalized scour depth. To assess the performance of the ANFIS model, observed equilibrium scour depth values were plotted against the predicted ones. Fig. 5 illustrates the results with the performance indices between predicted and observed data for the validating (testing) data sets. The result of the grouped variables data shows a high coefficient of determination ($R^2=0.98$) and also RMSE (=0.0795), and in the case of ANN-RBF has ($R^2=0.827$, RMSE=0.0988) in both training and validation periods but this variation is low compared with R^2 variation (Table 6). The results of Liriano and Day (2001) also are interesting but did not produce any implicit information for general use in designs and conventional regression-based equations (Lim 1995, Chiew and Lim 1996; Abt et al. 1984) estimates either under- or overpredict scour depth, only Lim's equation gives $R^2=0.53$ and Chiew and Lim's formula produces $R^2=0.45$ for testing data set. This study is useful for applications of culvert scour for field conditions because the ANFIS model was developed with wide range of data, which could be deemed as the closest to field conditions, particularly helping to identify parameters that most likely define scour processes and explain scour variability, and ANFIS model is shown to agree well with actual measurements.

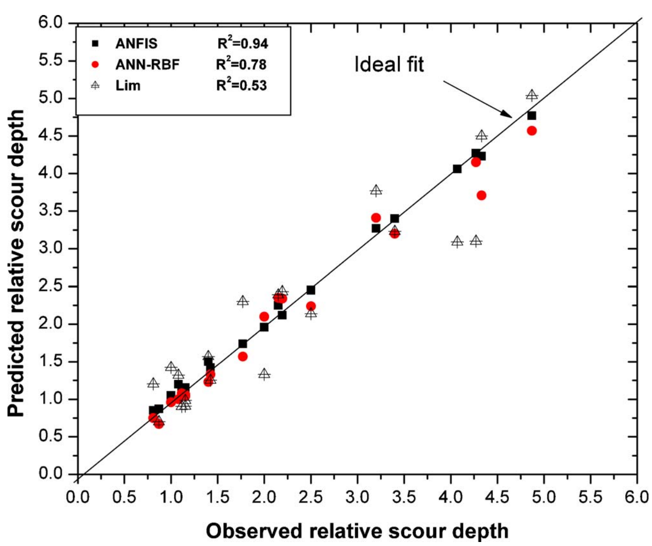

Fig. 5. Observed versus predicted scour depth—validation (testing)

Conclusions

The application of the soft computing approaches ANN-RBF and ANFIS to predict the local scour depth at culvert outlets was described. The ANFIS and ANN-RBF models were developed to predict the values of relative scour depth (ds/d_0) from laboratory measurements compiled from the literature. ANFIS-based approach was presented to estimate depth of scour at culvert outlet from optimum data sets. The application of the ANFIS in this study is another important contribution to scour-depth estimation methodologies for culverts. The present study indicates that employing the original data set yielded a network that can predict measured depth scour at culvert outlets more accurately than traditional regression analysis based formulas (Lim 1995; Chiew and Lim 1996; Abt et al. 1984) that under- and overpredict scour depths. The overall performance of ANFIS model is superior to the ANN model when compared to error based criteria. Further work is required to collect field data of scour at culvert outlets to train the GP approach and validate its usefulness.

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Notation

The following symbols are used in this paper:

d_s = maximum scour depth;
d_0 = pipe diameter for circular outlets and the outlet height for noncircular outlets;
d_{50} = particle mean diameter;
g = gravitational acceleration;
H = depth of water in the downstream receiving channel (tailwater depth);
K_s = shape of culvert;
R^2 = coefficient of determination;

u_0 = mean velocity at the outlet;
W = width of the receiving channel;
W_0 = width of the outlet;
α = spread;
δ = average absolute deviation;
μ = fluid dynamic viscosity;
μ_0 = dynamic viscosity of water;
ρ = fluid density;
ρ_s' = buoyant sediment density; and
σ_g = geometric standard deviation.

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