Alternative neural networks to estimate the scour below spillways

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Abstract

Artificial neural networks (ANN's) are associated with difficulties like lack of success in a given problem and unpredictable level of accuracy that could be achieved. In every new application it therefore becomes necessary to check their usefulness vis-à-vis the traditional methods and also to ascertain their performance by trying out different combinations of network architectures and learning schemes. The present study was oriented in this direction and it pertained to the problem of scour depth prediction for ski-jump type of spillways. It evaluates performance of different network configurations and learning mechanisms. The network architectures considered are the usual feed forward back propagation trained using the standard error back propagation as well as the cascade correlation training schemes, relatively less used configurations of radial basis function and adaptive neuro-fuzzy inference system. The network inputs were characteristic head and discharge intensity over the spillways while the output was the predicted scour depth at downstream of the bucket. The performance of different schemes was tested using error criteria of correlation coefficient, average error, average absolute deviation, and mean square error. It was found that the traditional formulae of Veronese, Wu, Martins and Incyth as well as a new regression formula derived by authors failed to predict the scour depths satisfactorily and that the neuro-fuzzy scheme emerged as the most satisfactory one for the problem under consideration. This study showed that the traditional equation-based methods of predicting design scour downstream of a ski-jump bucket could better be replaced by one of the soft computing schemes.

Keywords: Ski-jump scour; ANN; ANFIS; RBF; Error criteria; Scour depths

1. Introduction

Applications of neural network (ANN) to solve problems in water resources have been in vogue since last decade – although they are mostly confined to hydrology [1–3]. Employment of the ANN in solving hydraulics-oriented problems is relatively sparse and typically ranges from the work of Trent et al. [4] dealing with the sediment transport in open channels, Grubert [5], pertaining to the flow conditions under interfacial mixing in stratified estuaries to that of Nagy et al. [6], where the sediment discharge in rivers was predicted. Within the larger field of hydraulics, again a few investigators have addressed the uncertain issue of scour around structures with the help of the ANN. Examples of the latter studies include Trent et al. [7], who evaluated scour at bridge piers, Liriano and Day [8], who predicted depths of scour at culvert outlets, Kambekar and Deo [9], who estimated the scour geometry around groups of piles in the ocean and Azinfar et al. [10], who applied the ANN to forecast scour depths at the sluice gate.

The use of artificial neural networks as well as that of hybrid systems like the neuro-fuzzy seems to have been preferred by different investigators over conventional schemes likes non-linear regression and numerical methods due to so many relative advantages. Important ones among them are that the physics or mechanics of the underlying process need not be known beforehand, no mathematical model needs to be assumed a priori, and contrary to conventional analytical schemes the ANN is neither required to omit a large number of input variables nor use them after making simplification or specifying upper or lower bounds. Further the ANN's do not call for any exogenous input other than the input–output patterns for calibration and unlike many analytical or numerical models they are less dependent on the designer's expertise. It is however noted that the neural networks may have their own limitations in directly addressing and understanding physics of the underlying process and as such they may not completely replace existing mathematical or physical modeling.

Currently the ANN's are also associated with difficulties like lack of success in a given problem and unpredictable level of accuracy that could be achieved. It therefore becomes necessary that their usefulness vis-à-vis the traditional methods are checked for every new application and their performance is ascertained by trying out different combinations of network architectures and learning schemes. The present study is oriented in this direction. It differs from the previous works on the hydraulic scour referred to earlier in that it pertains to the scour at the ski-jump bucket type of spillways and uses field measurements rather than the controlled laboratory ones involved in most of the earlier studies to train the networks and also evaluates performance of different network configurations and learning mechanisms. The network architectures considered are the regular feed forward (FF) trained using the standard error back propagation (FFBP) as well as the cascade correlation (FFCC) training schemes, the relatively less used configurations of radial basis function (RBF) and adaptive neuro-fuzzy inference system (ANFIS). This paper is an updated and revised version of the conference paper [11].

2. The networks

A neural network represents interconnection of neurons, each of which basically carries out the task of combining the input, determining its strength by comparing the combination with a bias (or alternatively passing it through a non-linear transfer function) and firing out the result in proportion to such a strength as indicated below:

$$O = 1/[1 + e^{-S}] \tag{1}$$

$$S = (x_1 w_1 + x_2 w_2 + x_3 w_3 + \cdots) + \theta \tag{2}$$

where O = output from a neuron; $x_1, x_2, \ldots$ = input values; $w_1, w_2, \ldots$ = weights along the linkages connecting any two neurons and indicating strengths of the connections; θ = bias value. Eq. (1) indicates a transfer function of Sigmoid nature, commonly used; although there are other forms available, like sinusoidal, Gaussian, hyperbolic tangent. Textbooks like Kosko [12] and Wasserman [13] give theoretical details of the working of an ANN. The known input–output patterns are first used to train a network and strengths of interconnections (or weights) and bias values are accordingly fixed. Thereafter the network becomes ready for application to any unseen real world example. A supervised type of training involves feeding input–output examples till the network develops its generalization capability while an unsupervised training would involve classification of the input into clusters by some rule. In the supervised training the network output is compared with the desired or actual one and the error or the difference so resulted is processed through a mathematical algorithm. Normally such algorithms involve an iteration process to continuously change the connection weights and bias till the desired error tolerance is achieved. The traditional training method is the standard back-propagation, although numerous training schemes are available to impart better training with the same set of data, as indicated by Londhe and Deo [14] in their harbour tranquility studies.

Most of the previous works on ANN applications to water resources have included the feed forward type of the architecture, where there are no backward connections, (Fig. 1) trained using the error back propagation scheme or the FFBP configuration. The RBF network (Fig. 2) is also similar to this in that it has three layers of neurons, namely input, hidden and output. However it uses only one hidden layer, each neuron in which operates as per the Gaussian transfer function, as against the Sigmoid function of the common FFBP. Further while training of the latter is fully supervised (where both input–output examples are required), the same of the former is fragmented, wherein unsupervised learning of the input information, first classifies it into clusters, which in turn are used to yield the output after a supervised learning. This 'local tuning' could not only be more efficient but also more satisfactory in modeling data non-linearities than the common FFBP. Appendix I give mathematical expressions associated with the use of the RBF architecture.

The ANFIS on the other hand is a hybrid scheme which uses the learning capability of the ANN to derive the fuzzy if-then rules with appropriate membership functions worked out from the training pairs leading finally to the inference [15,16]. The difference between the common neural network and the ANFIS is that while the former captures the underlying dependency in the form of the

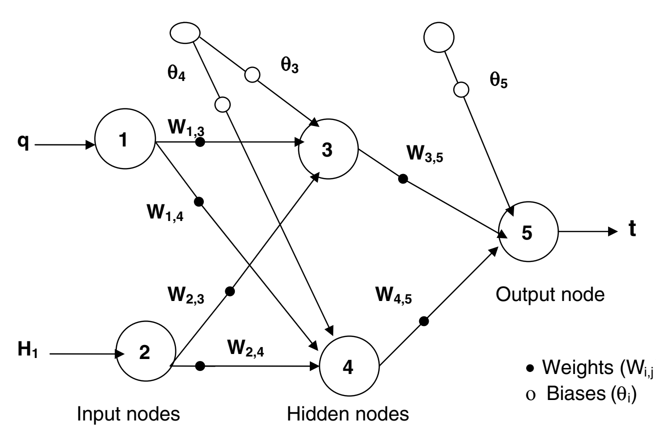

Fig. 1. The FFBP architecture.

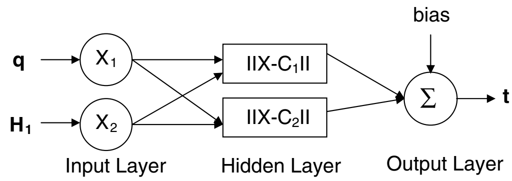

Fig. 2. RBF neural network architecture.

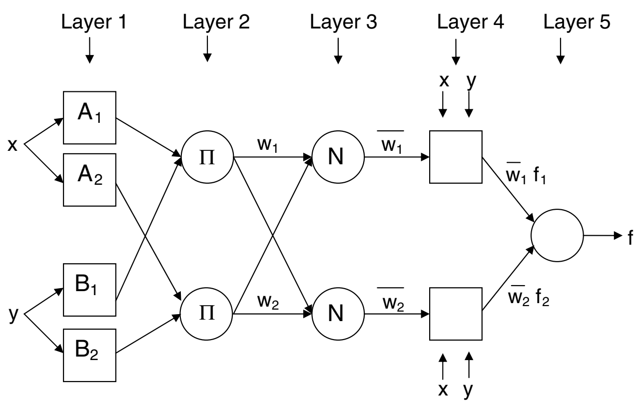

Fig. 3. ANFIS network architecture.

trained connection weights, the latter does so by establishing the fuzzy language rules. The input in ANFIS (Fig. 3) is first converted into fuzzy membership functions, which are combined together, and after following an averaging process, used to obtain the output membership functions and finally the desired output. The treatment of data non-linearities in this way has been recently found to be useful in fields like hydrology [17], traffic engineering [18] and soil analysis [19]. Mathematical expressions involved in the working of the ANFIS are given in Appendix I.

As mentioned earlier three different network architectures, namely common feed forward back propagation (FFBP), radial basis function (RBF) and adaptive neuro-fuzzy inference system (ANFIS) were considered. In order to ensure that the training by the back-propagation was adequate the feed forward network was further trained using two alternative schemes, namely back-propagation (BP) and cascade correlation (CC). The corresponding networks were termed as FFBP and FFCC. Thirumalaiah and Deo [20,21] had earlier indicated that efficient learning was possible when the cascade correlation method was used. Use of the neural network tool box under the Matlab software has been made in the present study. The function: 'newrb' involved in it provides an efficient design of the RBF and hence the same was employed herein. Similarly the 'genfis2' code that generates the first order Sugeno fuzzy system based on the subtractive clustering of data sets has been used to develop the ANFIS system.

3. The scour problem

Spillways provide for disposal of flood water in excess of the reservoir capacity and also lead to the control of water flow at the downstream. Out of several types of spillways the over-fall, ogee and breast wall spillways are more commonly used. The energy dissipation in such spillways can be in the form of ski-jump jet, which throws the water jet away from the bucket lip into the air, and then in the plunge pool formed at the point of impact on the tail water (Fig. 4). The impact of the high velocity jet gives rise to the scour both upstream and downstream of the point of impingement. Such impact is transmitted through cracks and fissures of the rock by way of hydrodynamic pressure fluctuations causing hydraulic jacking action and also by the transient pressure fluctuation caused due to air locking. This causes the rock mass to break into small pieces and to consequently get swept away in the downstream of the river. The erosion continues up to the point where the impinging jet energy is insufficient to exert breaking pressure on the rock or where the secondary current produced are less strong to remove the rock blocks [22].

The process of scouring continues till an equilibrium scour depth is reached, which corresponds to a situation where increased water depth in the scour hole precludes exertion of bed shear stress that is sufficient to cause further bed erosion or to a condition where rate of bed erosion is balanced by the rate of deposition of material brought back into the scour hole by the return flow. There are various hydraulic, morphologic and geotechnical factors governing the depth of scour, t, namely, (referring to Fig. 4) discharge intensity q, height of fall H_1, bucket radius R, bucket lip angle, ϕ, type of rock, degree of rock homogeneity, time and mode of operation of spillway. Various investigators over a period of several decades in the past have given empirical formulae based on laboratory as well as prototype observations in order to predict the scour depth downstream of the ski-jump bucket spillway. For Indian applications the formulae proposed by Veronese [23], Wu [24], Martins [25] and Incyth [26] are popular. The use of the Veronese formula has also been recommended in The Bureau of Indian standards [27] and is still followed in India. The expressions of these four formulae are respectively:

$$t = 1.90q^{0.54}H_1^{0.225} \tag{3}$$

$$\frac{t}{H_1} = 2.11\left(\frac{q}{\sqrt{qH_1^3}}\right)^{0.51} \tag{4}$$

$$t = 1.5q^{0.6}H_1^{0.1} \tag{5}$$

$$t = 1.42q^{0.50}H_1^{0.25} \tag{6}$$

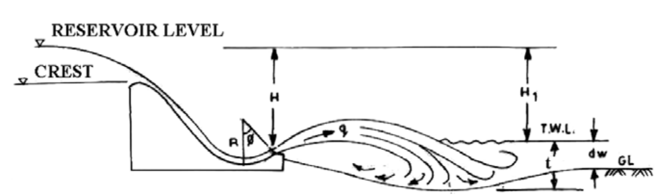

Fig. 4. Spillway and scour hole notations.

Since recent past the prediction of scour holes using numerical models has been attempted by some investigators. In such schemes the governing differential equations representing continuity of mass, momentum or energy and modeling turbulent flows or transport phenomenon through Navier–Stokes or Reynolds convection–diffusion equations are solved using a suitable numerical scheme. The corresponding studies are due to Hoffmans and Booij [28], who simulated development of the local scour, Olsen and Melaaen [29] and Olsen and Kjellesvig [30] who modeled the scour process around cylinders, Karim and Ali [31] and Ali et al. [32] , who predicted scour below free fall jets, Link and Zanke [33], who studied scour around bridge piers. Numerical studies involving scour around ski-jump bucket explicitly are however lacking and use of the empirical Eqs. (3)–(6) as above is till traditionally followed, especially in countries like India (as dictated by the design code), where a very large number of such spillways exist.

Use of the above equations is very convenient, however their major drawback is that they involve idealization, approximation and averaging of widely varying prototype conditions and could predict scour depths which may be considerably different than their actual values. e.g. the Rana Pratap sagar dam built across the River Chambal in India had design discharge intensity of 47.6 $m^3/s/m$ and corresponding head H_1 of 26.6 m. With these causative factors one would expect scour depths 't' of 32.01 m, 12.21 m, 21.1 m and 19.51 as per Eqs. (3)–(6) respectively. As against this the actual deepest scour observed was only 24.7 m. (incidentally the authors-derived Eq. (7) mentioned later also predicts $t = 20.67$ m, further indicating inadequacy of the regression approach). Eqs. (5) and (6) were presented later than Eqs. (3) and (4) by probably considering additional information available at that time and hence seem to yield estimates more close to the actual measurement. It is felt that such vast differences are partly due to the complexity of the phenomenon involved and partly because of the limitation of the analytical tool commonly used by most of the earlier investigators namely, non-linear statistical regression. The present study therefore reanalyzes the past data using the ANN's.

4. The database used

A majority of past works on scour predictions utilized the hydraulic model studies, which were more helpful in exploring the scour mechanism than in obtaining more accuracy in the depth estimation. They suffer from the problems arising out of the scale effects, inability to correctly model certain field conditions like bed morphology and loss of flow energy in aeration as well as failure to consider a variety of causative factors simultaneously. In this study therefore neural networks were calibrated with the help of realistic field conditions only, although it is recognized that prototype measurements may also suffer from instrumental uncertainties and inaccuracies and lack of availability of data on all causative parameters. The publications reporting such observations indicated that only three types of information, namely, scour depth below tail water level t, discharge intensity q, and head drop H_1 are uniformly reported in all references and that the information on other factors affecting the scour was not commonly available across them. Although there are many factors that affect the scour depth some of them only are of primary importance [34]. In addition considering that many traditional prediction formulae, including those due to Veronese [23], Wu [24], and Martins [25] are based only on q and H_1 a neural network with 2 input nodes and one output node only was developed (Fig. 1). In total there were 91 input–output pairs formed from the published data reported in Wu [24], Martins [25], Sen [35], Spurr [36], Wang [37], Akhmedov [38], Khatsuria [39], Yildiz and Üzücek [40] and Yildiz and Üzücek [41]. They are graphically shown in Fig. 5 which shows the ordinates of the observed scour depths against the varying values of q and H_1. Presence of a wide scatter and absence of fixed or regular and simple relationships between these input–output variables can be noted, which justifies application of the ANN's for the prediction problem under consideration.

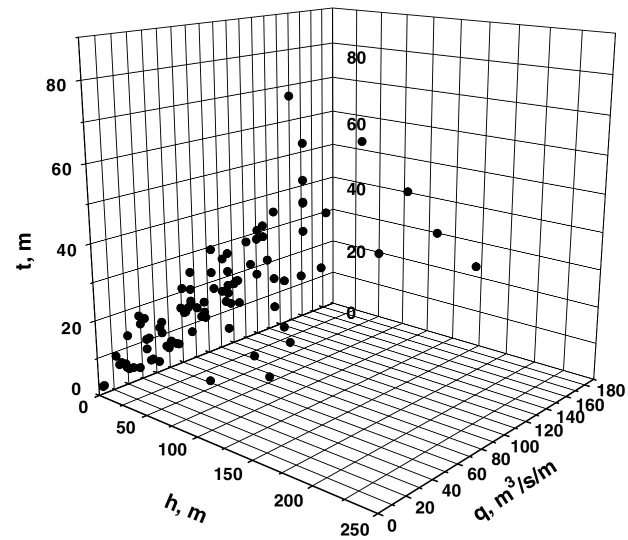

Fig. 5. Observed scour depths against varying values of q and H_1.

5. Network testing

Eighty percent input–output patterns, chosen randomly till the best training performance was seen, were used for network training while remaining ones were used for testing or validating the trained network. The number of input and output nodes for all the networks considered in this work was 2 and 1 respectively. The number of hidden nodes in FFBP was decided by trials. The network was trained by increasing the number of hidden nodes, starting from 1 and every time it's testing performance was noted on the testing set of data in terms of the error measures. The most acceptable testing performance was reached when the number of hidden nodes was only 2 in the present case (see Fig. 1). The number of hidden nodes for the RBF gets determined in the mathematical training process and this was also 2 in the present problem.

Figs. 6–9 show the testing results of the two FFBP and FFCC networks as well as those of the RBF and ANFIS networks respectively in terms of scatter plots of predicted versus observed scour. Such testing was also performed in respect of the traditional formulae proposed in the past by Veronese [23], Wu [24], Martins [25], Incyth [26] as well as in respect of a new regression equation derived by the authors on the basis of the compiled data set. The fit of the non-linear regression equation by authors to the training data set yielded following equation:

$$t = 1.42q^{0.44}H_1^{0.3} \tag{7}$$

Figs. 10–14 indicate respectively the corresponding formulae-based testing plots.

A quantitative comparison is shown in Table 1 in terms of the four error measures namely, (i) the correlation coefficient, r, which presents the degree of linear association between predicted and true values, (ii) the average error (+ or −), AE, which is a parameter commonly understood in engineering applications, and which considers algebraic difference between predicted and true values, (iii) the average absolute deviation, δ, which does not even out positive or negative errors as in AE, and (iv) the mean square error, MSE, which is preferred to in many iterative prediction and optimization schemes. Expressions for these measures are given in Appendix II.

The above exercise indicated that the values of r across different neural networks varied from 0.90 to 0.95; those of AE changed from −8.90% to −33.56% while the δ values ranged from 12.09% to 24.12%. In contrast to these net-

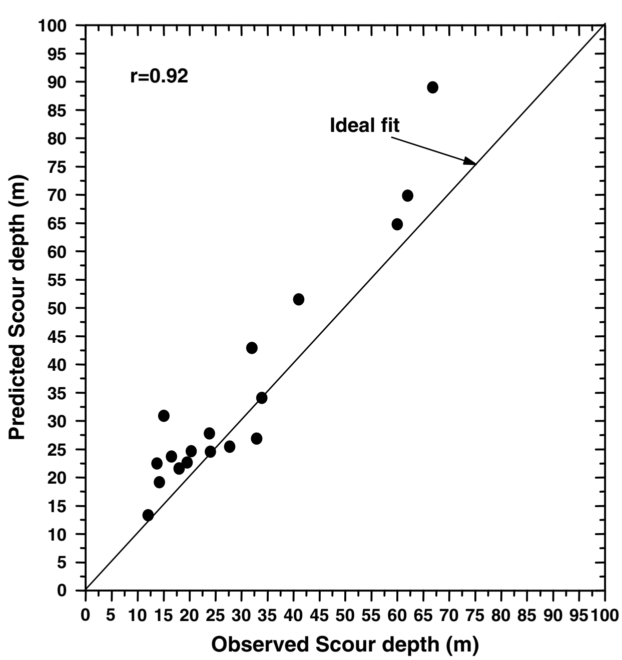

Fig. 6. Observed versus predicted scour depths by FFBP.

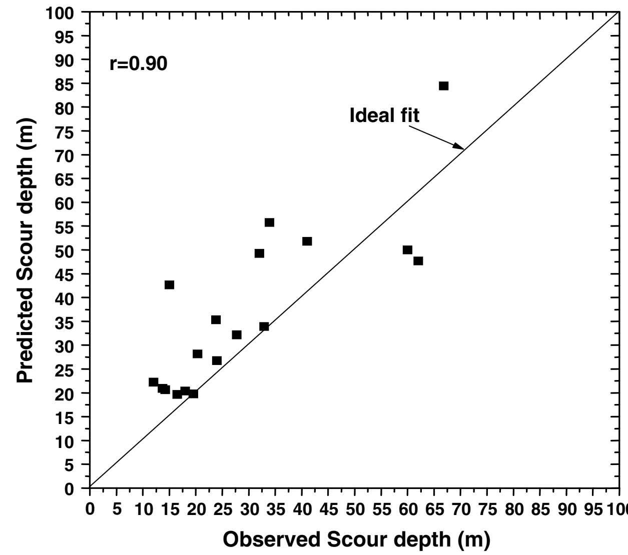

Fig. 7. Observed versus predicted scour depths by FFCC.

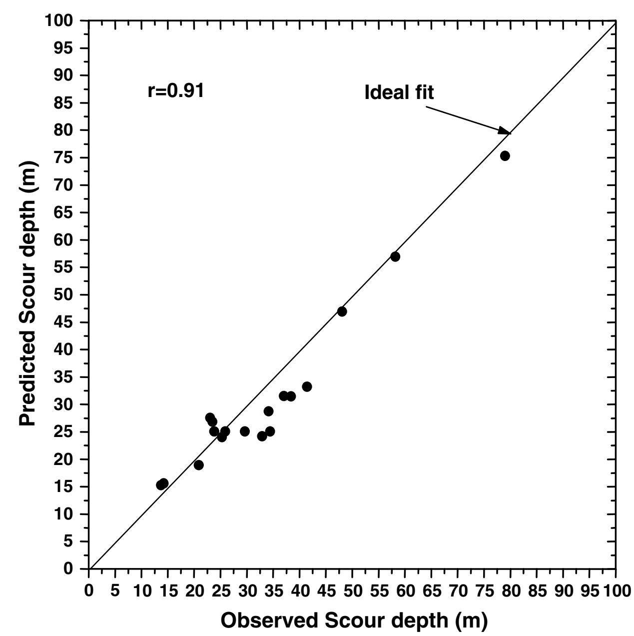

Fig. 8. Scatter plot of observed versus predicted scour depths by RBF.

work-based error measures, the values of r were much lower and those of AE and δ were much higher in the case of all regression-based formulae including the new non-linear regression formula. The FFBP and FFCC (Figs. 6 and 7) showed a tendency to overestimate, indicating that theses schemes were unable to model the data non-linearity adequately, unlike the RBF and the ANFIS schemes (Figs. 8 and 9). Table 1 suggests that the RBF and the ANFIS yielded more or less similar predictions (although RBF (Fig. 8) produced larger estimates in a better way than ANFIS (Fig. 9)), which is understandable considering similarities in the data processing with these methods. Fig. 7 indicates that the cascade correlation is only an efficient

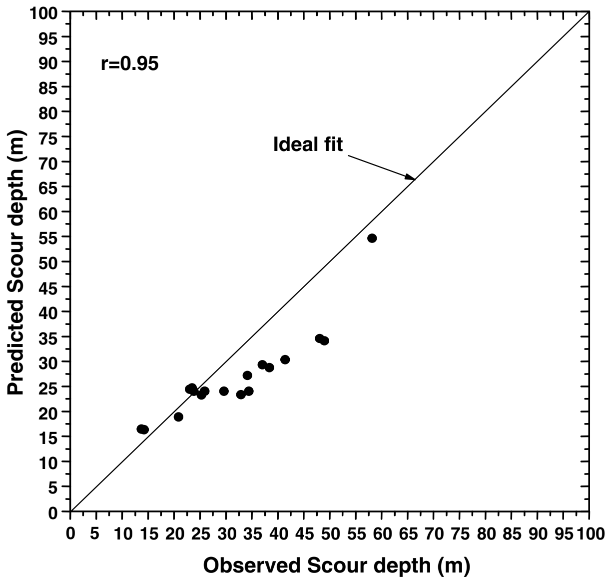

Fig. 9. Observed versus predicted scour depths by ANFIS.

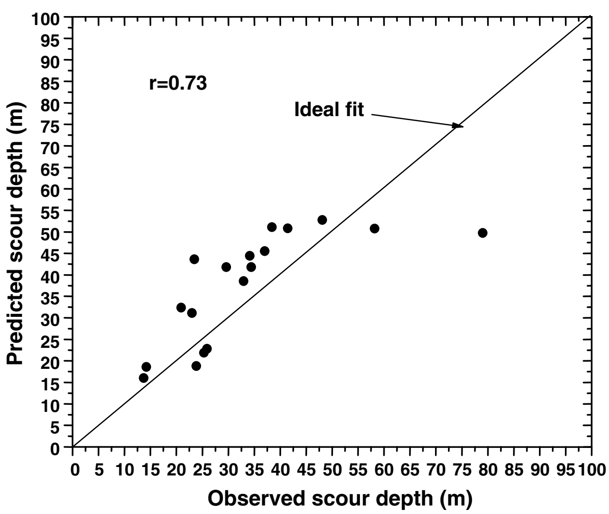

Fig. 10. Observed versus predicted scour depths by Veronese.

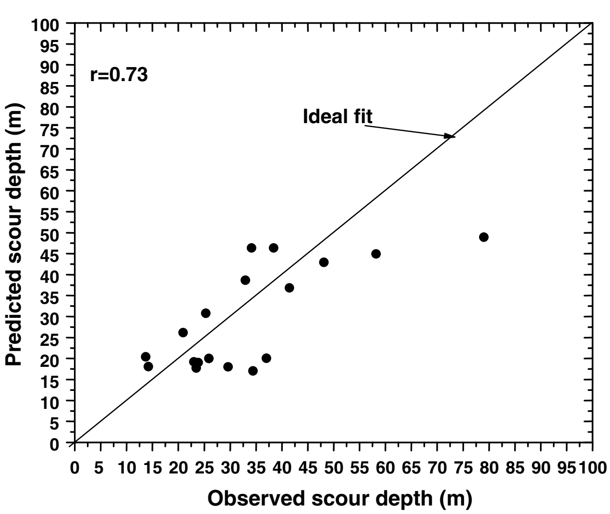

Fig. 11. Observed versus predicted scour depths by Wu.

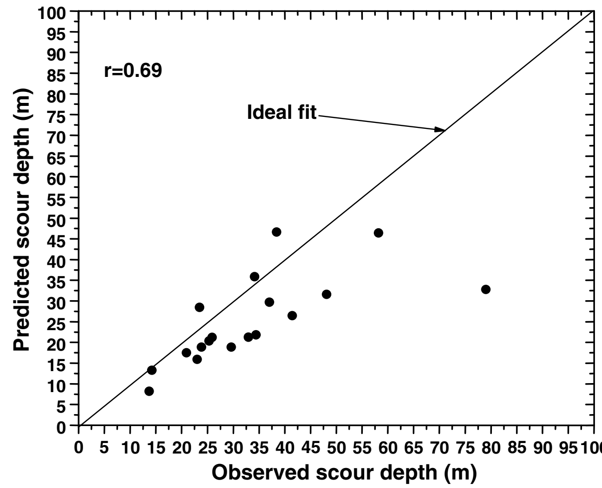

Fig. 12. Observed versus predicted scour depths by Martins.

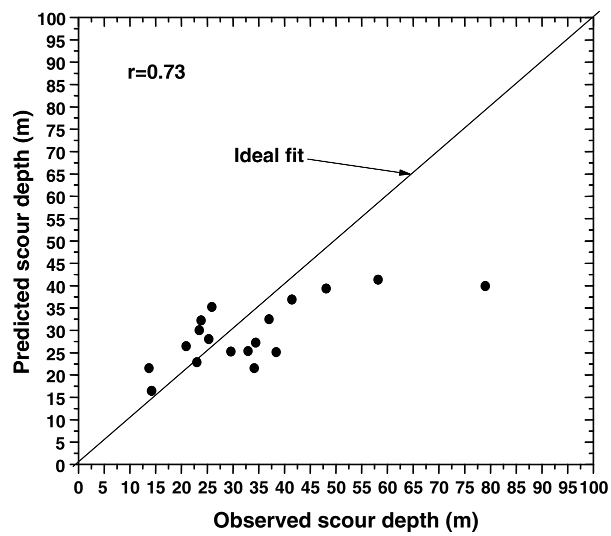

Fig. 13. Observed versus predicted scour depths by Incyth.

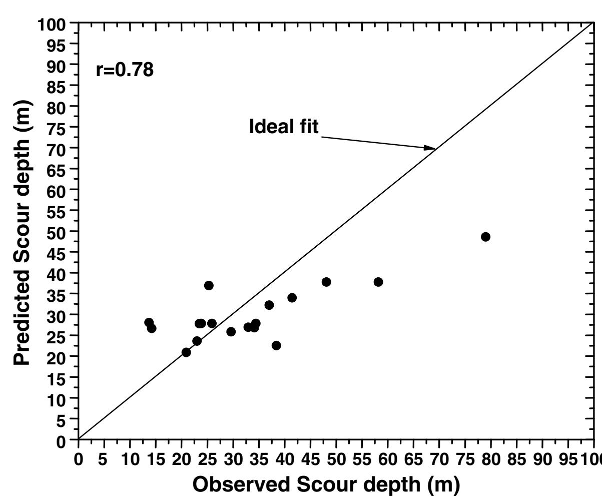

Fig. 14. Observed versus predicted scour depths by regression.

Table 1
Comparison of network – yielded and true scour depths

Figure no.	Method	Correlation coefficient, r	Average error, AE	Average absolute deviation, δ	Mean square error, MSE
6	Neural network (FFBP based)	0.92	−8.89	13.27	115.64
7	Neural network (FFCC based)	0.90	−33.56	24.12	154.79
8	RBF	0.91	10.02	19.93	64.68
9	ANFIS	0.95	13.64	12.09	55.43
10	Veronese	0.73	−18.50	22.57	130.02
11	Wu	0.73	16.94	27.60	128.92
12	Martins	0.69	21.15	28.52	193.85
13	Incyth	0.73	21.90	26.76	149.85
14	Regression (authors)	0.78	−24.90	26.89	136.78

method of training but it need not result in better accuracy compared with the ordinary back-propagation. Comparison of Fig. 7 with other similar figures shows that adequate training is really necessary while applying the neural network; otherwise one may not get results better than the traditional regression.

Among the five formulae listed earlier in Eqs. (3)–(7) the author-derived formula goes very close to that of the Incyth equation. A small change in the exponent of q and H_1 (in the Incyth formula) seems to result in improved predictions as reflected in higher 'r' and generally lower mean square error in the author-derived formula (see Figs. 13 and 14 and Table 1). The Veronese formula, suggested by the Indian design codes [27] showed a large error margin with respect to the neural networks. Among the prevailing regression formulae the Veronese' (Fig. 10) predictions are mostly higher, especially in the middle range, while Martins's (Fig. 12) are lower and so also of Wu's (Fig. 11), especially at higher scour levels. The Incyth, formula (Fig. 13) over-predicts at lower values and under-predicts at higher values. The author-derived Eq. (7) (Fig. 14) also suffers from the same problem but it produced the highest correlation coefficient among the regression equations. The databases used by the earlier investigators were different than the present one, which could be a major reason for such a discrepancy.

Table 1 clearly indicates overall best performance of the ANFIS in that it has the lowest values of all other error measures, except that of 'r' which only indicated relatively weak tendency of predictions to linearly co-vary with the actual values. The hybrid technique of ANFIS is the latest addition to the soft tools made available to civil engineers for soft computations and is believed to combine advantages of both ANN and fuzzy logic. While predicting certain sand mixture properties, Akbulbut et al. [19] had earlier also observed that the ANFIS-based predictions were better than the FFBP, although Nayak et al. [17] had noted to the contrary that when overall error criteria were applied ANFIS performed similar to the ANN in their riverflow prediction problem. It appears that the treatment to nonlinearities in the scour data meted out in the present problem by the ANFIS approach worked better than the other schemes. In other words the scour data seem to be more amenable to fuzzy if-then rules rather than crisp-value processing in the other networks. The ANFIS ensures localized functioning of the transfer function as against the globalized one of a general FFBP. This results in smaller number of values participating in the mapping process, which in turn requires limited data for training (as against the FFBP).

This study thus showed that traditional equation-based methods of predicting the design scour downstream of a ski-jump bucket could better be replaced by the neural network and similar soft computing schemes. Within the different networks employed the relatively advanced ANFIS could produce more satisfactory results.

The developed networks need not be always viewed as only 'black-boxes' or 'transfer functions'. Like regression based equations they can also be used to understand the underlying physical process to some extent. e.g. Azamathulla et al. [42] have shown how in case of data retrieved from the hydraulic model experiments the network can draw parametric variations of the scour depth with unit discharge, head and other causative variables for ski-jump spillways. Similarly Azamathulla [43] further made attempts to decipher the internal functioning of such networks. By studying the output fired by the hidden and the output neuron in a relative manner for varying scour depths it was concluded that the hidden neurons may individually model effects of a particular causative variable, may carry out a piece wise regression or may do some kind of partitioning of the input domain into sub-domain as a part of their modeling process. Wilby et al. [44] and Jain et al. [45] had earlier noticed similar facts while working on time series modeling in rainfall–runoff processes. Such an analysis in case of the present network showed that both the hidden neurons model the input–output process equally, which is understandable due to very small number of input and hidden neurons.

It is recognized that the above analysis is based on a limited amount of available records of scour depths (although all the networks involved were parsimonious). It would be interesting to know if the same level of accuracy can be achieved when the sample size increases indefinitely.

The present studies were based only on the field measurements for which knowledge of limited causal variables was available. A more general network however would involve consideration of additional input like hydraulic parameters (R and Φ) and geotechnical factors (rock size, rock mass rating, rock quality designation). This is discussed separately in Azamathulla et al. [42]. However it may be noted that the current studies presented networks based on actual field situations rather than on controlled laboratory conditions and hence might reflect reality in a better way. The work reported in this paper goes beyond that in Azamathullah et al. [46] that dealt with only the usual feed forward back propagation type of the network.

6. Conclusions

If the prediction of scour downstream of a ski-jump bucket is desired to be made with the help of the regression formulae alone then the new Eq. (7) derived by the authors based on compilation of past field data can be recommended in preference to the traditional equations by Veronese, Wu, Martins and Incyth. Among these prevailing formulae the Veronese equation over-predicts the actual scour while the Wu and Martins formulae under predict the same.

The usual feed forward networks of FFBP and FFCC showed a tendency to overestimate at lower values, unlike the RBF and the ANFIS. The RBF and the ANFIS yielded more or less similar predictions. A common application of the four different error criteria indicated an overall best performance of the ANFIS in this particular mapping problem. The treatment to non-linearities in the scour data meted out by the ANFIS approach worked much better than the other schemes. The scour data thus seem to be more amenable to fuzzy if-then rules rather than crisp-value processing. The ANFIS ensures localized functioning of the transfer function as against the globalized one of a general FFBP resulting in smaller number of values participating in the mapping process, and hence may work well, as in the present case, with limited data for training.

This study thus showed that the traditional equation-based methods of predicting design scour downstream of a ski-jump bucket could better be replaced by the neural network and similar soft computing schemes.

Appendix I. The RBF and ANFIS networks

The RBF network

The output y of a RBF network corresponding to input $\mathbf{x}$ is computed by the equation:

$$y = f(x) = \sum_{i=1}^{n} w_i R_i(x) + \theta \tag{8}$$

where w_i = connection weight between the hidden neuron and output neuron; θ = bias, $R_i(x)$ are radial basis functions given by (Fig. 2):

$$R_i(x) = \varphi\|x - c_i\| \tag{9}$$

having a maximum value at the origin that decays rapidly as its argument tends to infinity. It approaches zero as the Euclidean distance increases between an input vector and the center increases. The general class of radial basis functions is Gaussian:

$$R_i = -\exp\left(-\sum_{i=1}^{n} \frac{\|x_i - c_i\|^2}{2\sigma_{ij}^2}\right) \tag{10}$$

where $c_i^{\mathrm{T}} = [c_{i1}, c_{i2}, c_{i3} \ldots, c_{in}]$ is the center of the receptive field; and σ_{ij} = width of the Gaussian function which indicates the selectivity of the neuron. The major task of RBF network design is to determine center $\mathbf{c}$. The simplest and easiest way may be to choose the centers randomly from the training set. The second approach is to use the k-means technique of clustering input training set into groups and choose the center of each group as the center. Also, $\mathbf{c}$ can be treated as a network parameter along with w_i and adjusted through error-correction training. After the center is determined, the connection weights w_i between the hidden layer and output layer can be determined simply through ordinary back-propagation training.

The ANFIS network

This network (Fig. 3) works as follows: Let x and y be the three typical input values fed at the two input nodes, which will then transform those values to the membership functions (say bell-shaped) and give the output as follows: Note in general, w = output from a node; μ = membership function, A_i , B_i = fuzzy sets associated with nodes x, y.

$$\mu_{Ai}(x) = \frac{1}{1 + |(x - c_1)/a_1|^{2b_1}} \tag{11}$$

where a_1, b_1, and c_1 = changeable premise parameters. Similar computations are carried out for the input of y to obtain $\mu_{B_i}(y)$. The membership functions are thereafter multiplied in the second layer e.g.:

$$w_i = \mu_{Ai}(x).\mu_{B_i}(y) \quad (i = 1, 2) \tag{12}$$

Such products or firing strengths are then averaged, i.e.,

$$\overline{w_i} = w_i / \sum w_i \quad (i = 1, 2) \tag{13}$$

Nodes of the fourth layer use the above ratio as a weighting factor and using fuzzy if-then rules produce the output as below: (An example of the if-then rule is: If x is A_1 and y is B_1 then $f_1 = p_1 x + q_1 y + r_1$)

$$\overline{w_i} f_i = \overline{w_i}(p_i x + q_i y + r_i) \tag{14}$$

where p, q, r = changeable consequent parameters. The final network output f is produced by the node of the fifth layer as summation of all incoming signals, exemplified in the previous Eq. (14).

For imparting faster training and adjusting the network parameters to the above network a two-step process is used. In the first step the premise parameters are kept fixed and the information is propagated forward in the network to layer 4, where a least-squares estimator identifies the consequent parameters. In the second step, the backward pass, the consequent parameters are held fixed while the error is propagated, and the premise parameters are modified using the gradient descent. Apart from the training patterns the only user-specified information required is the number of membership functions for each input. The description of the learning algorithm is given in Jang and Sun [15].

Appendix II. The error measures

Correlation coefficient (r),

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}} \tag{15}$$

where $x = (X - \overline{X})$, $y = (Y - \overline{Y})$, X = observed values, $\overline{X}$ = mean of X, Y = predicted value, $\overline{Y}$ = mean of Y. The summation in the above equation as well as in the following two equations is carried out over all 'n' number of testing patterns.

Average error (AE),

$$\mathrm{AE} = \frac{\sum \frac{X-Y}{X} * 100}{n} \tag{16}$$

Mean square error (MSE),

$$\mathrm{MSE} = \frac{\sum (X-Y)^2}{n} \tag{17}$$

Average absolute deviation, δ:

$$\delta = \frac{\sum |(Y-X)|}{\sum X} * 100 \tag{18}$$

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