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Evaluation of GMDH networks for prediction of local scour depth at bridge abutments in coarse sediments with thinly armored beds

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ABSTRACT

Protection of the bridge abutment in waterways against scour phenomena is a very significant issue in hydraulic engineering fields. Several field and experimental investigations were carried out to produce a relationship between the abutment scour depth due to thinly armored bed and the governing variables. However, existing empirical equations do not always provide accurate scour prediction due to the complexity of the scour process. In the present study, group method of data handling (GMDH) networks are utilized to predict abutments scour depth in thinly armored beds. GMDH network is developed using evolutionary and iterative algorithms included those of gravitational search algorithm (GSA), particle swarm optimization (PSO), and back propagation (BP). The sediment size properties, bridge abutments geometry, and approaching flow are considered as effective parameters on the abutment scour depth. Training and testing stages of the models are carried out using experimental data sets. Performances results for alternative GMDH networks are compared with those obtained using traditional equations. A sensitivity analysis is also performed to determine the most important parameter in predicting the abutment scour depth in thinly armored beds.

1. Introduction

Local scour around bridge abutments is known as scour process because flow conditions changes by the abutments presence. Recently, investigations of abutment scour indicated which the scour at bridge abutments has very complex mechanism. The flow structure that cause abutment local scour is complex in details. This structure can be included down flow, primary vortex, secondary vortex, and wake vortices. The scour phenomena in non-cohesive bed materials are classified into the clear-water and live-bed conditions (Yakoub, 1995). In the last few decades, a large number investigations of abutment scour were carried out in non-cohesive soils (e.g., Dey et al., 2008; Dey and Barbhuiya, 2004; Richardson et al., 1993; Melville, 1992; Froehlich, 1989; Laursen, 1980; Liu et al., 1961).

A few researchers applied armored layer to decrease the scour depth around abutments. In fact, armored layer is defined as a protective surface-layer whose particles size is bigger than bed materials. The armored bed is composed of a coarse sediment that overlain on a bed with relatively fine sediment. The armored bed develops the value of critical shear velocity for the inception motion of bed materials and it is caused to increase stability of surface particles and structures. In addition, it was used to protect piers and abutments against scour process where are embedded in river or sea (Dey and Barbhuiya, 2004; Froehlich, 1995; Ettema, 1980; Melville, 1975). Dey and Barbhuiya (2004) carried out abutment scour experiments with thinly armored bed. They performed experiments with different geometry of bridge abutments in clear-water conditions. Dey et al. (2008) concluded that the dune height is the most important factor to cause the maximum damage of riprap. Through the experiments, empirical equations based on non-linear regressions were yielded from their investigations. Empirical equations are restricted to the range of experimental database. Therefore, it should be noted that empirical equations in term of traditional methods based regression have not highly generalization capacity to apply for designing the scour depth around abutments due to armored beds.

In this way, various artificial intelligence approaches such as artificial neural network (ANN), adaptive neuro-fuzzy inference system (ANFIS), genetic programming (GP), and linear genetic programming (LGP) were used to predict scour depth around hydraulic structures (Azamathulla et al., 2014; Zahiri et al., in press; Dehghani et al. 2013; Azamathulla, 2012a, 2012b; Azamathulla et al., 2011, 2010, 2008a, 2008b; Guven and Gunal

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2008; Azmathullah et al., 2005). Among these methods, the GMDH network is known as a self-organized method to model and forecast the behaviors of unknown or complicated systems based on given input–output data pairs (Amanifard et al., 2008). In addition, the GMDH approach is used in different fields of engineering sciences such as energy conservation, control engineering, system identification, marketing, economic, and geology (Mehrara et al., 2009; Kalantary et al., 2009; Amanifard et al., 2008; Srinivasan, 2008; Witczak et al., 2006). In fact, the main advantage of the GMDH model is to build analytical functions within feed forward network based on quadratic polynomial whose weighting coefficients are obtained using regression method (Kalantary et al., 2009). Recently, the GMDH networks were used to predict the scour depth around hydraulic structures. Performances result showed that these approaches can be provided more accurate scour depth prediction than those obtained using other artificial intelligence approaches and traditional methods (Najafzadeh and Barani, 2011; Najafzadeh and Azamathulla, 2013a, 2013b; Najafzadeh et al., 2013a, 2013b, 2013c, 2013d, 2014).

The main objective of this study is to investigate the efficiency of the GMDH networks for predicting the abutment scour with armored bed. The GMDH network is developed using GSA, PSO, and BP algorithms. Performances of the proposed approaches are compared with those yielded using traditional methods.

1.1. Data presentation

Local scour depth around abutments with armored beds depends on properties of sediment size, abutments geometry, and characterization of approaching flow (e.g., Dey et al., 2008; Dey and Barbhuiya, 2004). Therefore, the effective parameters on the abutment scour can be expressed as follows:

$$d_{sa} = f(U_{ca}, \rho_s, g, l, \rho, h, t, d_a, d) \tag{1}$$

where d_{sa}, U_{ca}, ρ_s, g, l, ρ, h, t, d_a, and d are the scour depth due to armored bed, critical velocity for armor-layer particles, mass density of sediments, acceleration due to gravity flow depth, length of abutment, mass density of water, approaching flow depth, thickness of armor-layer, medium diameter of armor-layer particles, and medium diameter of bed sediments, respectively.

Using dimensional analysis, group of dimensionless parameters was resulted as follows:

$$d_{sa}/l = f(K_s, F_{ca}, h/l, t/d_a, d_a/d) \tag{2}$$

where F_{ca} is the critical abutment Froude number. The F_{ca} is defined as follows:

$$F_{ca} = U_{ca}/\sqrt{(\rho_s/\rho - 1).g.l} \tag{3}$$

In addition, K_s is the abutment shape factor. K_s value depends on abutments geometry. In this study, abutment shape factor being 1 for vertical-wall abutments, 0.82 for 45° wing-wall abutments, and 0.75 for semicircular abutments (Dey and Barbhuiya, 2004). Details of abutments geometry are given in Table 1. Also, schematic sketches of abutments are illustrated in Fig. 1. General configuration of scour process at an abutment in an thinly armored layer was illustrated in Fig. 2. In Fig. 2, δ parameter is depth of secondary armor-layer in scour hole. In case of applications of artificial intelligence models to evaluate local scour depth, using of grouped dimensionless parameters indicated better predictions of scour depth than that of dimensional parameters (e.g., Najafzadeh and Barani, 2011; Azamathulla et al., 2010; Guven and Gunal, 2008a). In this way, Eq. (2) is used to develop the GMDH networks. Data sets were collected from Dey and Barbhuiya (2004) experiments. Ranges of input and output parameters used for scour modeling are given in Table 2. Out of a total of 99 data sets, about 75% (74 data sets) are selected randomly for training, whereas the remaining 25% (25 sets) are used to test models.

Table 1
Dimensions of abutments used in the scour depth modeling.

Designation	Vertical-wall abutment		45° wing-wall abutment		Semicircular-wall abutment	
	$l(m)$	$b(m)$	$l(m)$	$b(m)$	$l(m)$	$b(m)$
Type 1	0.06	0.12	0.06	0.18	0.06	0.12
Type 2	0.08	0.16	0.08	0.24	0.08	0.16
Type 3	0.1	0.2	0.1	0.3	0.1	0.2

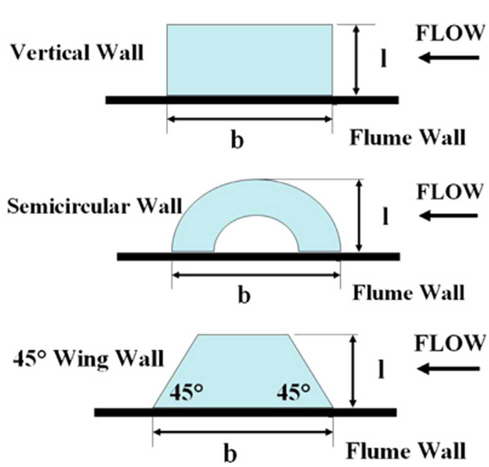

Fig. 1. Different types of abutments used for the scour depth modeling.

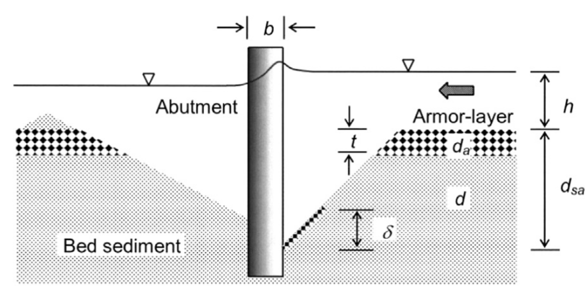

Fig. 2. General configuration of scour process at an abutment in an thinly armored layer (Dey and Barbhuiya, 2004).

Table 2
Ranges of input and output parameters used to develop the GMDH networks.

Parameters	Ranges
h	0.099–0.154 (m)
l	0.06–0.1 (m)
d	0.26–0.91 (mm)
b	0.12–0.3 (m)
d_a	1.15–5.45 (mm)
t	4–15 (mm)
U_{ca}	0.0197–0.909 (m/s)
ρ	1000 (kg/m^3)
ρ_s	2650 (kg/m^3)
d_{sa}	0.103–0.283 (m)
g	10 (m/s^2)
K_s	0.75–1

1.2. Models descriptions

In this section, details of the GMDH network are discussed. Also, mechanism development of GMDH network using the PSO, GSA, and BP algorithms are presented.

2. Group method of data handling (GMDH)

The GMDH network is a learning machine based on the principle of heuristic self-organizing, proposed by Ivakhnenko in the 1960s. Also, it is a series of operations of seeding, rearing, crossbreeding, selection and rejection of seeds correspond to the determination of the input variables, and structure and parameters of model, and selection of model by principle of termination (Madala and Ivakhnenko, 1994). The GMDH network is a very flexible algorithm and it can be hybridized by other evolutionary algorithms, such as genetic algorithm (Mehrara et al., 2009; Amanifard et al., 2008), genetic programming (Najafzadeh and Barani, 2011; Iba and de Garis, 1996), particle swarm optimization (Onwubolu, 2008), levenberg-marquardt (Najafzadeh et al., 2013c), and back propagations (Najafzadeh and Azamathulla, 2013a, 2013b; Najafzadeh and Barani, 2011; Srinivasan, 2008; Sakaguchi and Yamamoto, 2000). Previous researches established that hybridizations were successful in finding solutions to problems in different fields.

By means of the GMDH network, a model can be represented as a set of neurons in which different pairs of them in each layer. These neurons are connected through a quadratic and triquadratic polynomial and thus produce new neurons in the next layer. Such representation can be used in modeling from map inputs to outputs. The formal definition of system identification problem is to find a function $\hat{f}$ that can be approximately used instead of actual function f, in order to predict the output $\hat{y}$ for a given input vector $X=(x_1,x_2,x_3,...,x_n)$ as close as possible to its actual output y. Therefore, given n observation of multi-input-single-output data pairs so that

$$y_i=f(x_{i1},x_{i2},x_{i3},...,x_{in})\ (i=1,2,\ldots,\mathrm{M}) \tag{4}$$

It is now possible to train the GMDH network to predict the output values $\hat{y}_i$ for any given input vector $X=(x_{i1},x_{i2},x_{i3},...,x_{in})$, that is

$$\hat{y}_i=\hat{f}(x_{i1},x_{i2},x_{i3},...,x_{in})\ (\mathrm{i}=1,2,\ldots\mathrm{M}). \tag{5}$$

In order to solve this problem, the GMDH builds the general relationship between output and input variables in the form of mathematical description, which is also called reference.

The problem is now determining the GMDH network so that the square of difference between the actual output and the predicted one is minimized, that is

$$\sum_{i=1}^{M}\left[\hat{f}(x_{i1},x_{i2},x_{i3},....,x_{in})-y_i\right]^2\to \min. \tag{6}$$

General connection between inputs and output variables can be expressed by a complicated discrete form of the Volterra function, a series in the form of:

$$y=w_0+\sum_{i=1}^{n}w_ix_i+\sum_{i=1}^{n}\sum_{j=1}^{n}w_{ij}x_ix_j+\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}w_{ijk}x_ix_jx_k+...,\tag{7}$$

which is known as the Kolmogorov–Gabor polynomial (Sanchez et al., 1997; Farlow, 1984; Ivahnenko, 1971). In the present study, quadratic polynomial of the GMDH network is used that is written as:

$$\text{Quadratic}:\hat{y}=G(x_i,x_j)=w_0+w_1x_i+w_2x_j+w_3x_ix_j+w_4x_i^2+w_5x_j^2 \tag{8}$$

This network of connected neurons builds the general mathematical relation of inputs and output variables given in Eq. (7). The weighting coefficients of Eq. (8) are calculated using regression techniques (Farlow, 1984) so that the difference between actual output, y, and the calculated one, $\hat{y}$, for each pair of $x_i$and x_j as input variables is minimized. Indeed, it can be seen that a tree of polynomials is constructed using the quadratic form given in Eq. (8) whose weighting coefficients can be obtained by least-squares sense. In this way, the weighting coefficients of quadratic function G_i are obtained to optimally fit the output in the whole set of input–output data pairs, that is

$$E=\frac{\sum_{i=1}^{M}(y_i-G_i())^2}{M}\to \min. \tag{9}$$

In the basic form of the GMDH algorithm, all the possibilities of two independent variables out of total n input variables are taken in order to construct the regression polynomial in the form of Eq. (7) that best fits the dependent observations $(y_i,i=1,2,...,M)$ in a least-square sense. Consequently, $C_n^2=n(n-1)/2$ neurons of quadratic polynomial will be built up in the first layer of the feed forward network from observations $\{(y_i,x_{ip},x_{iq});(i=1,2,...M)\}$ for different $p,q\in\{1,2,....n\}$. In other words, it is now possible to construct M data triples $\{(y_i,x_{ip},x_{iq});(i=1,2,...M)\}$ from observation using such $p,q\in\{1,2,\ldots.n\}$ in the form

$$\begin{bmatrix} x_{1p} & x_{1q} & y_1\\ x_{2p} & x_{2q} & y_2\\ x_{mp} & x_{mq} & y_m\end{bmatrix}. \tag{10}$$

Using the quadratic sub-expression in the form of Eq. (7) for each row of M data triples, the following matrix equation can be readily obtained as:

$$AW=Y \tag{11}$$

where W is the vector of unknown weighting coefficients of the quadratic polynomial in Eq. (8)

$$W=\{w_0,w_1,w_2,w_3,w_4,w_5\}^T \tag{12}$$

The superscript T represents transpose of matrix.

$$Y=\{y_1,y_2,y_3,...,y_M\}^T \tag{13}$$

is the vector of observation values of outputs. It can be readily seen that

$$A=\begin{bmatrix}1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2\\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2\\ 1 & x_{mp} & x_{mq} & x_{mp}x_{mq} & x_{mp}^2 & x_{mq}^2\end{bmatrix} \tag{14}$$

The least-squares technique from multiple-regression analysis leads to the solution of the normal equations in the form of:

$$W=(A^TA)^{-1}A^TY \tag{15}$$

which determines the vector of the best weighting coefficients of the quadratic Eq. (7) for the whole set of M data triples. It should be noted that this procedure is repeated for each neuron of the next hidden layer according to the connectivity topology of the network (Mehrara et al., 2009). A schematic diagram of the GMDH network was depicted in Fig. 3.

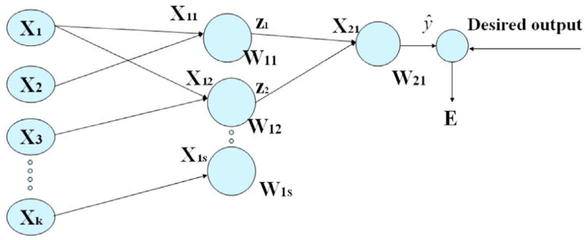

Fig. 3. General structure of the GMDH network.

3. Application of PSO algorithm in the topology design of GMDH network

The Particle swarm optimization (PSO) was inspired by the social behavior of animals such as fish schooling, insects swarming and birds flocking. PSO was introduced by Kennedy and Eberhart (2001) in the mid-1990s, to simulate the graceful motion of bird swarms as a part of a socio-cognitive study. It involves a number of particles that are initialized randomly in the search space of an objective function. These particles are referred to as swarm. Each particle of the swarm represents a potential solution of the optimization problem. The ith particle in tth iteration is associated with a position vector, X_i^t, and a velocity vector, V_i^t, that shown as following:

$$X_i^t = \{x_{i1}^t, x_{i2}^t, ..., x_{iD}^t\} \tag{16}$$

$$V_i^t = \{v_{i1}^t, v_{i2}^t, ..., v_{iD}^t\} \tag{17}$$

where D is dimension of the solution space.

The particle fly through the solution space and its position is updated based on its velocity, the best position particle (***pbest***) and the global best position (***gbest***) that swarm has visited since the first iteration as,

$$\boldsymbol{V}_i^{t+1} = \omega^t \boldsymbol{V}_i^t + c_1 r_1 (\boldsymbol{pbest}_i^t - \boldsymbol{X}_i^t) + c_2 r_2 (\boldsymbol{gbest}^t - \boldsymbol{X}_i^t) \tag{18}$$

$$\boldsymbol{X}_i^{t+1} = \boldsymbol{X}_i^t + \boldsymbol{V}_i^{t+1} \tag{19}$$

where r_1 and r_2 are two uniform random sequences generated from interval [0, 1]; c_1 and c_2 are the cognitive and social scaling parameters, respectively and ω^t is the inertia weight that controls the influence of the previous velocity.

Shi and Eberhart (1998) proposed that the cognitive and social scaling parameters c_1 and c_2 should be selected as $c_1 = c_2 = 2$ to allow the product $c_1 r_1$ or $c_2 r_2$ to have a mean of 1. The performance of PSO is very sensitive to the inertia weight (ω) parameter which may decrease with the number of iteration as follows (Shi and Eberhart, 1998; Salajegheh et al., 2008, 2009):

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{t_{\max}}.t \tag{20}$$

where $\omega_{\max}$ and $\omega_{\min}$ are the maximum and minimum values of ω, respectively; and t_{max} is the limit numbers of optimization iteration. Performing the GMDH and PSO algorithms is a parallel action in each PD. The GMDH-PSO model has five input variables and one output. For the optimization of GMDH structure, the quadratic polynomial was selected as a second order polynomial. In addition, Eq. (8) was considered as an objective function.

Through the optimization process, the PSO algorithm optimized weighting coefficients of quadratic polynomial in each neuron. After that error values for each neuron is calculated using Eq. (9). Then, neurons are selected to generate the next layer. This process could be continued until minimum error of training network is obtained. The most significant feature of the GMDH-PSO algorithm is interaction between GMDH network and PSO algorithm. The other details of GMDH-PSO are presented in literatures (Onwubolu, 2008; Onwubolu and Sharma, 2004). Regarding this optimization process, a number of control parameters including range of constrain, c_1, c_2, number of particles, range of ω values, maximum iteration of optimization problem were considered as the values of the control parameters of the PSO algorithm. These parameters were presented in Table 3.

Table 3
Values of the PSO properties for prediction of the scour depth.

Parameter	Range
Omega	0.04–0.09
Number of particles	200
Number of variables	6
Maximum iteration	400
Error	0.0001
C_1 and C_2	1.5
Weighting coefficients	−0.5 to 1

Furthermore, from the GMDH-PSO network, corresponding polynomials representation for selective neurons of d_{sa}/l are as follows:

$$(d_{sa}/l)_1^1 = -0.174 + 0.4877K_s + 0.8634F_{ca} + 0.9882K_s.F_{ca} + 0.6999K_s^2 + F_{ca}^2 \tag{21}$$

$$(d_{sa}/l)_2^1 = 0.5627 + 0.047F_{ca} + 0.3577h/l + F_{ca}.h/l + 0.91575F_{ca}^2 - 0.03244(h/l)^2 \tag{22}$$

$$(d_{sa}/l)_5^1 = -0.21 + 0.38234K_s + 0.7835h/l + 0.9397K_s.h/l + 0.10873K_s^2 - 0.1591(h/l)^2 \tag{23}$$

$$(d_{sa}/l)_9^1 = 0.06488 - 0.0635F_{ca} + 0.02586d_a/d + 0.5286F_{ca}.d_a/d - 0.5F_{ca}^2 - 0.02852(d_a/d)^2 \tag{24}$$

$$(d_{sa}/l)_1^2 = 0.15818 + 0.6146(d_{sa}/l)_1^1 + 0.1837(d_{sa}/l)_2^1 - 0.0056(d_{sa}/l)_1^1 .(d_{sa}/l)_2^1 - 0.1087((d_{sa}/l)_1^1)^2 + 0.16431((d_{sa}/l)_2^1)^2 \tag{25}$$

$$(d_{sa}/l)_5^2 = 0.4563 + 0.07572(d_{sa}/l)_2^1 + 0.4246(d_{sa}/l)_9^1 + 0.15943(d_{sa}/l)_2^1 .(d_{sa}/l)_9^1 + 0.02396((d_{sa}/l)_2^1)^2 - 0.05948((d_{sa}/l)_9^1)^2 \tag{26}$$

$$(d_{sa}/l)_6^2 = 0.3548 + 0.7127(d_{sa}/l)_5^1 - 0.2295(d_{sa}/l)_9^1 + 0.8149(d_{sa}/l)_9^1 .(d_{sa}/l)_6^1 - 0.3395((d_{sa}/l)_9^1)^2 - 0.2798((d_{sa}/l)_6^1)^2 \tag{27}$$

$$(d_{sa}/l)_1^3 = 0.1838 + 0.09817(d_{sa}/l)_1^2 + 0.75739(d_{sa}/l)_5^2 + 0.23978(d_{sa}/l)_5^2 .(d_{sa}/l)_1^2 - 0.08547((d_{sa}/l)_5^2)^2 - 0.1274((d_{sa}/l)_1^2)^2 \tag{28}$$

$$(d_{sa}/l)_3^3 = -0.2248 + 0.58646(d_{sa}/l)_5^2 + 0.6342(d_{sa}/l)_6^2 + 0.09489(d_{sa}/l)_5^2 .(d_{sa}/l)_6^2 - 0.10057((d_{sa}/l)_5^2)^2 - 0.04138((d_{sa}/l)_6^2)^2 \tag{29}$$

$$(d_{sa}/l)_1^4 = -0.06372 + 0.32416(d_{sa}/l)_1^3 + 0.7235(d_{sa}/l)_3^3 + 0.05429(d_{sa}/l)_1^3 .(d_{sa}/l)_3^3 - 0.06021((d_{sa}/l)_1^3)^2 - 0.00161((d_{sa}/l)_3^3)^2 \tag{30}$$

In which superscript and subscript of each parameter present the number of pertaining layer and neuron, respectively.

3.1. Application of BP algorithm in the topology design of GMDH network

In this section, the learning method of the improved GMDH network is explained in brief. As one example, the following case is considered. In Fig. 3, x_v and z_s are the input and intermediate variables, respectively. W_{ts} denotes the weight vector. Furthermore, X_{ts} is the input vector for the neurons (t = number of layer and s = number of neuron in layer). These variables are defined as follows:

$$E = \frac{(y - \hat{y})^2}{2} \tag{31}$$

$$\hat{y} = W_{21}.X_{21} \tag{32}$$

$$z_1 = W_{11}.X_{11} \tag{33}$$

$$z_2 = W_{12}.X_{12} \tag{34}$$

$$W_{21} = \{w_{21}^0, w_{21}^1, w_{21}^2, w_{21}^3, w_{21}^4, w_{21}^5\}^T \tag{35}$$

$$W_{11} = \{w_{11}^0, w_{11}^1, w_{11}^2, w_{11}^3, w_{11}^4, w_{11}^5\}^T \tag{36}$$

$$W_{12} = \{w_{12}^0, w_{12}^1, w_{12}^2, w_{12}^3, w_{12}^4, w_{12}^5\}^T \tag{37}$$

each of weighting coefficients is corrected based on the back propagation method (Sakaguchi and Yamamoto, 2000; Najafzadeh and Barani, 2011). The partial differentiation is taken for the error function based on the chain rule, that is,

$$\frac{\partial E}{\partial W_{2s}} = \frac{\partial E}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial W_{2s}} \tag{38}$$

$$\frac{\partial E}{\partial W_{1s}} = \frac{\partial E}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial X_{2s}} \cdot \frac{\partial X_{2s}}{\partial z_s} \cdot \frac{\partial z_s}{\partial W_{1s}} \tag{39}$$

Thus, the learning laws are obtained as follows:

$$W_{2s}^{new} = W_{2s}^{old} + \eta . \frac{\partial E}{\partial \hat{y}} . X_{2s} \tag{40}$$

$$W_{1s}^{new} = W_{1s}^{old} + \eta . \frac{\partial E}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial X_{2s}} \frac{\partial X_{2s}}{\partial z_s} X_{1s} \tag{41}$$

where η is learning rate that is between 0 and 1.

As increasing the layer, the update rules corresponding to each layer are derived based on the same idea. The initial layer is simply the input layer. The first layer is created by computing regressions of the input variables and then choosing the best ones. The second layer is created by computing regressions of the values in the first layer along with the input variables. This means that the algorithm essentially builds polynomials of polynomials. Again, only the best of them are chosen by Eq. (8). This mechanism will be continued until a pre-specified selection criterion is met. In output layer, error of training network estimated by Eq. (9) and the new weighting coefficients are calculated using Eqs. (38)–(41). Again, output of each neuron is estimated from the first layer to output layer. This process is called feed forward and the correction of weighing coefficients of a network is called backward pass (Sakaguchi and Yamamoto, 2000; Srinivasan, 2008). Furthermore, from the GMDH-BP network, corresponding polynomials representation for selective neurons of d_{sa}/l are as follows:

$$(d_{sa}/l)_1^1 = 0.541 + 0.4164K_s - 0.09356F_{ca} + 2.17513K_s.F_{ca} - 1.0022K_s^2 - 0.97951F_{ca}^2 \tag{42}$$

$$(d_{sa}/l)_5^1 = 0.41034 - 0.14847K_s + 0.5848h/l + 0.2322K_s.h/l + 0.0288K_s^2 - 0.09417(h/l)^2 \tag{43}$$

$$(d_{sa}/l)_7^1 = -0.4234 + 1.4967h/l + 0.0181d_a/d + 1.3084(h/l).(d_a/d) - 0.8662(h/l)^2 - 0.5636(d_a/d)^2 \tag{44}$$

$$(d_{sa}/l)_1^2 = -45.208 + 109.022(d_{sa}/l)_1^1 - 7.957(d_{sa}/l)_5^1 + 9.388(d_{sa}/l)_1^1 .(d_{sa}/l)_5^1 - 63.2207((d_{sa}/l)_1^1)^2 + 2.679((d_{sa}/l)_5^1)^2 \tag{45}$$

$$(d_{sa}/l)_3^2 = -42.737 + 102.24(d_{sa}/l)_5^1 - 2.004(d_{sa}/l)_7^1 + 3.4359(d_{sa}/l)_5^1 .(d_{sa}/l)_7^1 - 59.076((d_{sa}/l)_5^1)^2 - 0.0914((d_{sa}/l)_7^1)^2 \tag{46}$$

$$(d_{sa}/l)_1^3 = -1.0827 + 1.8177(d_{sa}/l)_1^2 + 0.293(d_{sa}/l)_3^2 - 0.012(d_{sa}/l)_1^2 .(d_{sa}/l)_3^2 - 0.179((d_{sa}/l)_1^2)^2 - 0.0155((d_{sa}/l)_3^2)^2 \tag{47}$$

In which superscript and subscript of each parameter present the number of pertaining layer and neuron, respectively.

3.2. Application of GSA algorithm in the topology design of GMDH network

Recently, a novel heuristic search algorithm whish is called Gravitational Search Algorithm (GSA), has been proposed motivated by gravitational law and laws of motion. This algorithm has high performance in solving various optimization problems (Rashedi et al., 2009).

In GSA, a set of agents called masses are introduced to find the optimum solution by simulation of Newtonian laws of gravity and motion (Rashedi et al., 2011; Li and Zhou, 2011; Najafzadeh and Azamathulla, 2013a, 2013b; Najafzadeh and Lim, 2014). To describe the GSA consider a system with s masses in which the position of the ith mass is defined as follows:

$$X_i = (x_i^1, ..., x_i^d, ..., x_i^n) \quad , i = 1, 2, ..., s \tag{48}$$

where x_i^d is the position of ith mass in the dth dimension and n is dimension of the search space. The mass of each agent is calculated after computing current population's fitness as follows:

$$q_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{49}$$

$$M_i(t) = \frac{q_i(t)}{\sum_{j=1}^{s} q_j(t)} \tag{50}$$

where $M_i(t)$ and $fit_i(t)$ represent the mass and the fitness value of the agent i at t, and, $worst(t)$ and $best(t)$ are defined as follows (for a minimization problem):

$$worst(t) = \max_{j \in \{1,...,s\}} fit_j(t) \tag{51}$$

$$best(t) = \min_{j \in \{1,...,s\}} fit_j(t) \tag{52}$$

To compute the acceleration of an agent, total forces from a set of heavier masses should be considered based on gravity law (Eq. (53)). Also, it is followed by calculation of agent acceleration using motion law (Eq. (54)).

$$F_i^d(t) = \sum_{j \in kbest, j \neq i} rand_j G(t) \frac{M_j(t) M_i(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t)) \tag{53}$$

$$a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} = \sum_{j \in kbest, j \neq i} rand_j G(t) \frac{M_j(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t)) \tag{54}$$

Afterward, the next velocity of an agent is calculated as a fraction of its current velocity added to its acceleration (Eq. (55)). Then, its position could be calculated using Eq. (56).

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{55}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{56}$$

where $rand_i$ and $rand_j$ are two uniform random in the interval [0, 1], ε is a small value, and $R_{ij}(t)$ is the Euclidian distance between two agents i and j that were defined as $R_{ij}(t) = \|X_i(t), X_j(t)\|_2$. $kbest$ is the set of first K agents with the best fitness value and biggest mass. $kbest$ is a function of time, initialized to K_0 at the beginning

Table 4
Values of the GSA properties for predicting the scour depth.

Parameter	Range
Alpha	20
G_0	100
Number of variables	6
Maximum iteration	100
Error	0.00001
Number of agents	50
Weighting coefficients	−1 to 1

and decreasing with time. Here, K_0 is set to s (total number of agents) and is decreased linearly to 1. In GSA, the gravitational constant, G, will take an initial value, G_0, and it will be reduced by time:

$$G(t) = G(G_0, t) \tag{57}$$

In present study, we used Eq. (58) for the gravitational constant,

$$G(t) = G_0 e^{-\alpha \frac{t}{T}} \tag{58}$$

Through optimization process, values for number of agents, maximum number of iterations, α, and G_0 values were fixed. The values of the control parameters of the GSA algorithm were given in Table 4. In fact, GSA optimized weighting coefficients in each neuron of the GMDH network. Furthermore, from the GMDH-GSA network, corresponding polynomials representation for selective neurons of d_{sa}/l are as follows:

$$(d_{sa}/l)_1^1 = 0.5995 + 0.6095K_s + 0.76673F_{ca} + 0.74015K_s.F_{ca} - 0.2944K_s^2 + 0.94394F_{ca}^2 \tag{59}$$

$$(d_{sa}/l)_5^1 = 0.4358 + 0.51736K_s + 0.5559h/l + 0.33071K_s.h/l + 0.21K_s^2 - 0.08358(h/l)^2 \tag{60}$$

$$(d_{sa}/l)_9^1 = 0.3724 + 0.7799F_{ca} + 0.3179d_a/d + 0.17233(F_{ca}).(d_a/d) + 0.6874(F_{ca})^2 - 0.0242(d_a/d)^2 \tag{61}$$

$$(d_{sa}/l)_2^2 = -0.3542 + 0.48336(d_{sa}/l)_1^1 - 0.1338(d_{sa}/l)_5^1 + 0.22735(d_{sa}/l)_1^1 .(d_{sa}/l)_5^1 + 0.001213((d_{sa}/l)_1^1)^2 + 0.19606((d_{sa}/l)_5^1)^2 \tag{62}$$

$$(d_{sa}/l)_3^2 = 0.21042 + 0.002811(d_{sa}/l)_5^1 - 0.16977(d_{sa}/l)_9^1 + 0.18928(d_{sa}/l)_5^1 .(d_{sa}/l)_9^1 + 0.1585((d_{sa}/l)_5^1)^2 + 0.10564((d_{sa}/l)_9^1)^2 \tag{63}$$

$$(d_{sa}/l)_1^3 = -0.46056 + 0.76534(d_{sa}/l)_1^2 + 0.81063(d_{sa}/l)_3^2 + 0.2897(d_{sa}/l)_1^2 .(d_{sa}/l)_3^2 - 0.19399((d_{sa}/l)_1^2)^2 - 0.2961((d_{sa}/l)_3^2)^2 \tag{64}$$

In which superscript and subscript of each parameter present the number of pertaining layer and neuron, respectively.

4. Results and discussion

The performances of the GMDH networks for training and testing stages were presented in this section. In the models development, errors obtained in each selective neuron within the GMDH-BP, GMDH-PSO, and GMDH-GSA networks were indicated in Fig. 4. Also, proposed structures of the GMDH networks were illustrated in Figs. 5–7. For the GMDH-PSO model, Fig. 5 indicated 4, 3, 2, and 1 selective neurons (or polynomial neuron) in the first, second, third, and fourth layers, respectively. Fig. 6 illustrated selective polynomial neurons of the GMDH-BP model for prediction of local scour depth at abutments. Through the GMDH-BP development, 3 (1st, 5th, and 7th neurons), 2 (2nd and 3rd neurons), and 1 polynomial neurons were generated in the 1st, 2nd, and 3rd, respectively. Furthermore, Fig. 7 indicated structure of the GMDH-GSA model in form of 3, 2, and 1 selective neurons for the 1st, 2nd, and 3rd layers, respectively.

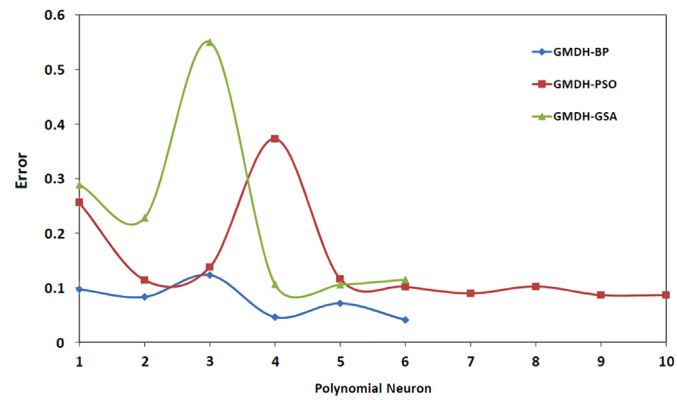

Fig. 4. Values of training errors related to the GMDH networks.

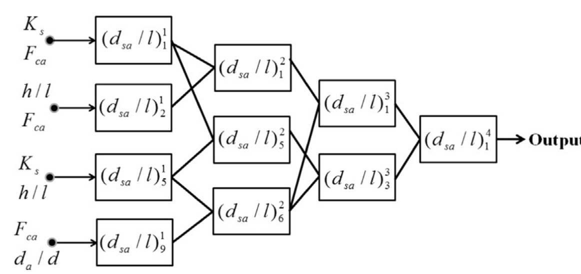

Fig. 5. Proposed structure of the GMDH-PSO network for abutment scour depth prediction.

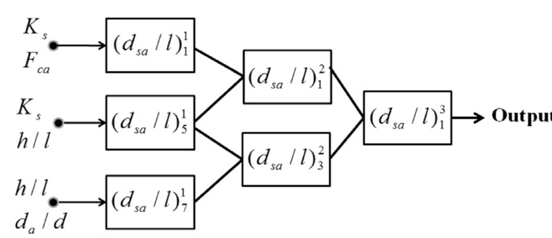

Fig. 6. Proposed structure of the GMDH-BP network for abutment scour depth prediction.

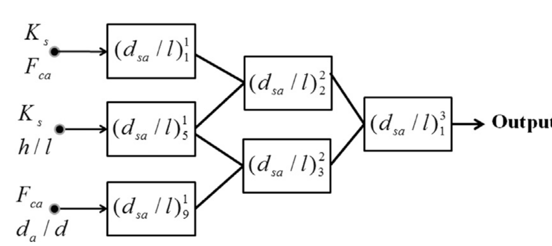

Fig. 7. Proposed structure of the GMDH-GSA network for abutment scour depth prediction.

In the present study, several traditional equations were utilized to evaluate abutment scour depth (Table 5). Performances of alternative GMDH networks are compared with those obtained traditional methods. Correlation coefficient (R), root mean square error (RMSE), and mean absolute percentage of error (MAPE),

Table 5
Traditional equations used to predict the abutment scour depth.

Traditional equations	Authors	Eq. no
$d_{sa}/l = 1.8K_s.(h/l)^{0.5}$	Lim (1997)	(65)
$d_{sa}/l = 2.27K_s.(\rho_s/\rho - 1)^{0.305}(h/l)^{0.265}F_c^{0.61} + h/l$	Hec-18 (Froehlich, 1989)	(66)
$d_{sa}/l = 7.273K_s.(\rho_s/\rho - 1)^{0.165}(h/l)^{0.265}F_c^{0.33}$	Hec-18 (Richardson et al., 2001)	(67)
$d_{sa}/l = 5.16K_s.(h/l)^{0.18}F_{ca}^{0.26}.(t/d_a)^{-0.19}.(d/d_a)^{-0.15}$	Dey and Burbhuiya (2004)	(68)

BIAS, and scatter index (SI) which are commonly used as indicator of errors prediction in training and testing stage (Najafzadeh et al., 2013d):

$$R = \frac{\sum_{i=1}^{M}\left[(d_{sa}/l)_{i(\text{Observed})} - \overline{(d_{sa}/l)}_{(\text{Observed})}\right]\left[(d_{sa}/l)_{i(\text{Predicted})} - \overline{(d_{sa}/l)}_{(\text{Predicted})}\right]}{\sqrt{\sum_{i=1}^{M}\left[(d_{sa}/l)_{i(\text{Observed})} - \overline{(d_{sa}/l)}_{(\text{Observed})}\right]^2 \cdot \sum_{i=1}^{M}\left[(d_{sa}/l)_{i(\text{Predicted})} - \overline{(d_{sa}/l)}_{(\text{Predicted})}\right]^2}} \tag{69}$$

$$RMSE = \left[\frac{\sum_{i=1}^{M}\left[(d_{sa}/l)_{i(\text{Predicted})} - (d_{sa}/l)_{i(\text{Observed})}\right]^2}{M}\right]^{1/2} \tag{70}$$

$$MAPE = \frac{1}{M}\left[\sum_{i=1}^{M}\left|\frac{(d_{sa}/l)_{i(\text{Predicted})} - (d_{sa}/l)_{i(\text{Observed})}}{(d_{sa}/l)_{i(\text{observed})}}\right| \times 100\right] \tag{71}$$

$$BIAS = \frac{\sum_{i=1}^{M}\left[(d_{sa}/l)_{i(\text{Predicted})} - (d_{sa}/l)_{i(\text{Observed})}\right]}{M} \tag{72}$$

$$SI = \frac{RMSE}{(1/M)\sum_{i=1}^{M}(d_{sa}/l)_{i(\text{Observed})}} \tag{73}$$

Performances results for training stages indicate that the GMDH-BP produced the scour depth prediction with lower error (RMSE=0.21 and MAPE=8.607) and higher coefficient correlation (R=0.96) compared to the GMDH-PSO and GMDH-GSA models. SI and BIAS obtained by the GMDH-BP were 0.113 and 0, respectively. Furthermore, the GMDH-PSO model provided relatively lower error of scour prediction (MAPE=13.14 and BIAS=0.005) than those obtained using the GMDH-GSA (MAPE=18.23 and BIAS=0.077). From statistical parameters of RMSE, MAPE, and BIAS, it can be said that GMDH-BP model is quantitatively superior to the GMDH-PSO and GMDH-GSA networks. Fig. 8 illustrates scatter plot of predicted values of scour depth using alternative GMDH networks versus those of observed values for training stages. Statistical error parameters for training stages were presented in Table 6.

From the models training, it can be said that GMDH-BP network provided a structure with fewer selective neurons (6 neurons) in comparison with the GMDH-PSO and GMDH-GSA models. In addition, volume of calculations for the GMDH-BP is lower than the GMDH-PSO and GMDH-GSA models.

Performances for testing stages of the GMDH networks indicated that the GMDH-PSO predicts the scour depth with relatively higher correlation coefficient (R=0.94) than the GMDH-BP (R=0.93) and GMDH-GSA (R=0.87) networks. Furthermore, the GMDH-BP model provided relatively lower error of scour predictions (RMSE=0.357 and MAPE=12.9) than the GMDH-PSO model (RMSE=0.388 and MAPE=14.39). Another interesting point is that, the GMDH-BP and GMDH-PSO networks provided same error of local scour prediction in term of SI parameter. BIAS values showed that GMDH-BP network (BIAS=0.11) produced relatively good prediction for the abutment scour depth, compared to the GMDH-PSO (BIAS=0.145) and GMDH-GSA (BIAS=0.175) models. Evaluation of statistical parameters for the GMDH networks demonstrated that combination of the GMDH and BP algorithms through the iterative process produced good performances in comparison with other models.

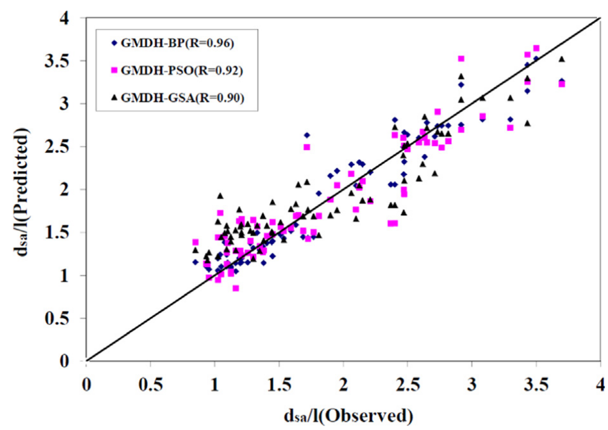

Fig. 8. Scatter plot of observed and predicted scour depth for training stages of the GMDH networks.

Table 6
Statistical results of the GMDH networks for training and testing stages.

	Training stage				
Network	R	RMSE	MAPE	BIAS	SI
GMDH-BP	0.96	0.21	8.607	0.00	0.113
GMDH-PSO	0.92	0.294	13.14	0.005	0.158
GMDH-GSA	0.90	0.341	18.23	0.077	0.183
	Testing stage				
Network	R	RMSE	MAPE	BIAS	SI
GMDH-BP	0.93	0.357	12.9	0.11	0.26
GMDH-PSO	0.94	0.388	14.38	0.145	0.224
GMDH-GSA	0.87	0.45	16.35	0.175	0.263

Fig. 9 illustrates scatter plot of predicted values of scour depth using alternative GMDH networks versus those of observed values for testing stages.

From Table 6, it can be found that performances of traditional equations produced quite higher error of scour prediction compared to the alternative GMDH networks. Lim (1997) equation predicted the abutment scour depth with lower error (RMSE=0.69, MAPE=20.31, and SI=0.39) in comparison with other traditional equations. BIAS error parameters related to the qualitative evaluation indicated that Lim (1997) equation has the quite lower over predictions than those obtained using other traditional equations. It is apparent that Eq. (66) provided the scour depth prediction with relatively higher accurate than those

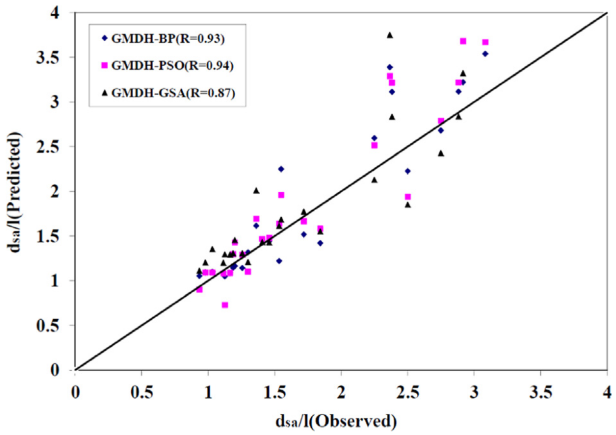

Fig. 9. Scatter plot of observed and predicted scour depth for testing stages of the GMDH networks.

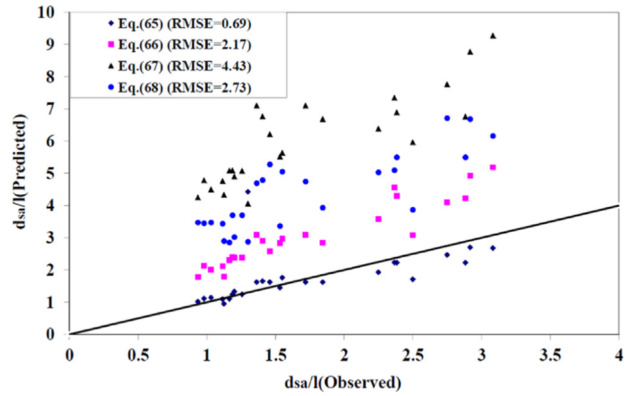

Fig. 10. Scatter plot of observed and predicted scour depth for evaluating the traditional equations.

Table 7
Statistical results of the traditional equations for evaluating the abutment scour depth.

Method	R	RMSE	MAPE	BIAS	SI
Eq. (65)	0.54	0.69	20.31	0.0366	0.39
Eq. (66)	0.45	2.13	103.52	1.59	1.23
Eq. (67)	0.84	4.4	273.94	4.31	2.54
Eq. (68)	0.81	2.74	168.97	2.64	1.58

traditional equations given by Eq. (67) (RMSE=4.4, MAPE=103.52, and BIAS=1.59) and Eq. (68) (RMSE=2.74, MAPE=168.97, and BIAS=2.64). Statistical results yielded by traditional equations for evaluating the abutment scour depth are given in Table 7. Fig. 10 illustrates scatter plot of predicted values of scour depth using empirical equations versus those observed values.

To clarify the new contributions of this study, efficiency of GMDH networks was investigated by three abutments shapes. In this way, performances of GMDH networks were presented for semicircular, vertical-wall, and 45° wing-wall abutments in Table 8. Results indicated that GMDH-BP network produced local scour depth around semicircular abutment with lower error of scour prediction (RMSE=0.125 and MAPE=8.24) than those obtained using the GMDH-PSO (RMSE=0.16 and MAPE=10.41) and GMDH-GSA (RMSE=0.26 and MAPE=20.223) models. For the vertical-wall abutment, the GMDH-BP network similar provided more accurate prediction than the other models. Also, performances of the GMDH networks for the 45° wing-wall abutments indicated that the GMDH-BP model (RMSE=0.57 and MAPE=22.07) predicted the scour depth with quite higher accuracy compared to the GMDH-PSO (RMSE=0.56 and MAPE=23.06) and GMDH-GSA (RMSE=0.95 and MAPE=43.87) models.

4.1. Sensitivity analysis

To determine the importance of each input variable on the scour depth, the GMDH-BP network was applied to perform a sensitivity analysis. The analysis is conducted such that, one parameter of Eq. (2) is eliminated each time to evaluate the effect of that particular input on output. Results of the analysis indicated that F_{ca} (R=0.85, RMSE=0.444, and SI=0.233) is the most effective parameter on the scour depth whereas the t/d_a (R=0.94, RMSE=0.233, and SI=0.14) has the least influence on scour depth for the GMDH-BP model, respectively. The other effective parameters on the d_s/l were K_s, h/l, and d_a/d, ranked from higher to lower values, respectively. Statistical error

Table 8
Comparison of results GMDH networks for different shapes of bridge abutments.

Method	Semicircular abutment	Vertical-wall abutment	45° Wing-wall abutment
GMDH-GSA	RMSE=0.26 MAPE=20.223	RMSE=0.534 MAPE=16.97	RMSE=0.95 MAPE=43.87
GMDH-PSO	RMSE=0.16 MAPE=10.41	RMSE=0.374 MAPE=11.76	RMSE=0.56 MAPE=23.06
GMDH-BP	RMSE=0.125 MAPE=8.24	RMSE=0.27 MAPE=10.75	RMSE=0.57 MAPE=22.07

Table 9
Statistical results of sensitivity analysis for the GMDH-BP.

Functions	R	RMSE	SI
$d_{sa}/l=f(F_{ca},h/l,t/d_a,d_a/d)$	0.91	0.35	0.194
$d_{sa}/l=f(K_s,h/l,t/d_a,d_a/d)$	0.851	0.444	0.233
$d_{sa}/l=f(K_s,F_{ca},t/d_a,d_a/d)$	0.91	0.336	0.17
$d_{sa}/l=f(K_s,F_{ca},h/l,d_a/d)$	0.94	0.233	0.14
$d_{sa}/l=f(K_s,F_{ca},h/l,t/d_a)$	0.94	0.26	0.14

parameters yielded from sensitivity analysis are given in Table 9. To obtained new contribution of this study, effects of models output on the variations of F_{ca} parameter were investigated. In this way, the discrepancy ratio (DR), known as the ratio of predicted and observed values, was utilized to quantify the sensitivity of the proposed models to F_{ca} parameter. A DR value of 1 shows a promisingly perfect agreement, while values greater (or smaller) than 1 indicate over (or under) prediction of the scour depth. Variations of DR values were plotted versus the logarithm of F_{ca}.

The result of the GMDH-BP model was illustrated in Fig. 11. The minimum, mean, and maximum DR values for the GMDH-BP model were obtained 0.77, 1.048, and 1.45, respectively. For $0.251<F_{ca}<0.851$, DR values decrease thereafter increase. Also, it indicates that GMDH-BP model has high over prediction of scour depth. For F_{ca}=0.037, Fig. 11 illustrates that the GMDH-BP model provides good agreement with the observed scour depth.

The results of GMDH-PSO model were shown in Fig. 12. The DR values were yielded between 0.65 and 1.389. Also, mean value of DR was 1.061. For $0.251<F_{ca}<0.851$, values of several points indicate that over (or under) predictions of the scour depth are met. Also, GMDH-PSO model produces relatively accurate scour depth because the scour depth values are trend to 1. The results of GMDH-GSA model were shown in Fig. 13. The DR values were yielded between 0.46 and 3.36. Also, mean value of DR was 1.23. For $0.251<F_{ca}<0.851$, several DR values become 1. For F_{ca}=0.037,

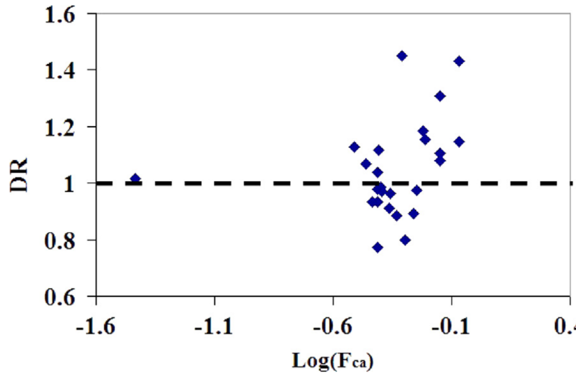

Fig. 11. Variation of DR with log(F_{ca}) for GMDH-BP model.

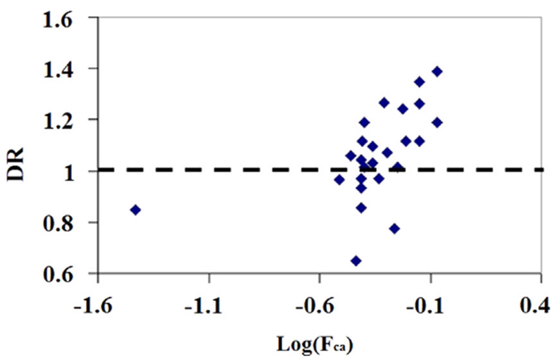

Fig. 12. Variation of DR with log(F_{ca}) for GMDH-PSO model.

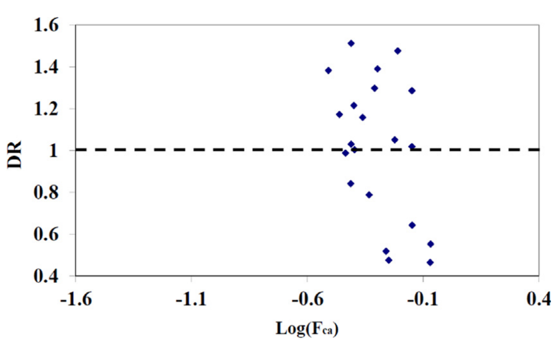

Fig. 13. Variation of DR with log(F_{ca}) for GMDH-GSA model.

GMDH-GSA model produces has remarkably higher under prediction of scour depth in comparison with the GMDH-BP and GMDH-PSO models.

5. Conclusion

In this study, abutment scour depth in thinly armored beds was predicted using alternative GMDH networks. The structure of GMDH network was trained using back propagation, particle swarm optimization, and gravitational search algorithm to determine weighting coefficients of quadratic polynomials. Data sets for performing the training and testing stages of alternative GMDH networks were collected from literature. Five inputs and one output parameters were assigned through the dimensional analysis for the abutment scour depth modeling due to thinly armored bed and clear water conditions. Furthermore, traditional equations given by Froehlich (1989), Lim (1997), Richardson et al. (2001), and Dey and Barbhuiya (2004) were used for comparisons.

Performing the proposed models for training stage indicated that GMDH-BP model produced relatively lower error (RMSE=0.21 and MAPE=8.607) and higher correlation coefficient (R=0.96) compared to those obtained using the GMDH-PSO and GMDH-GSA models. Through the testing stage, GMDH-BP model yielded better prediction with lower error (RMSE=0.357 and MAPE=12.9) rather than that obtained using performing the GMDH-PSO and GMDH-GSA models. From the traditional equations, Eq. (65) proposed by Lim (1997) predicted the abutment scour depth with compromisingly good agreements (RMSE=0.69 and MAPE=20.31) in comparison with other ones. In fact, Eq. (65) was validated for wider ranges of experimental datasets than those observed by Dey and Burbhuiya (2004). In general, traditional equations produced the scour depth with significantly higher error than those obtained using alternative GMDH networks. Results of sensitivity analysis indicated that F_{ca} is the most important parameters in modeling of scour depth by the GMDH-BP model. Also, through a brief comparative study, the GMDH-BP network provided good performances for semicircular, vertical-wall, and 45° wing-wall abutments compared to the GMDH-PSO and GMDH-GSA models.

In general, application of the evolutionary and iterative algorithms in topology design of the GMDH network proved that the GMDH network can be used successfully in prediction of scour problems in hydraulic engineering field.

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