Application of Particle Densimetric Froude Number for Evaluating the Maximum Culvert Scour Depth

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Abstract: This paper presents a compilation of a large database consisting of 588 culvert scour experiments covering a wide combination of different culvert shapes, culvert outlet conditions, and sediment properties. The analysis shows that the maximum scour depth, when normalized with the hydraulic radius, can be expressed reasonably as a linear function of the particle densimetric Froude number for both full-flowing and non-full-flowing outlet conditions. In addition, the sediment nonuniformity effect is also considered by choosing d_{84} (rather than d_{50}) as the representative grain diameter in the dimensional analysis, which improves the result of the data correlation. **DOI: 10.1061/(ASCE)IR.1943-4774.0001487.** *© 2020 American Society of Civil Engineers.*

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Introduction

Scour at a natural channel is caused by the removal of materials from the bed and banks by the flowing water, which happens naturally or as a result of construction of a hydraulic structure (May et al. 2002). Driven by the water jet ensuing from the culvert barrel, scour at the downstream of culvert outlets has long been recognized as a common engineering problem that can cause instability and damage to the culvert structure. Besides, excessive scour and channel degradation at the culvert outlet may cause drops in water surface, which, in turn, become a passage barrier for movement of aquatic species (Kilgore et al. 2010). The evaluation of scour dimension based on an empirical formula has been an important routine in hydraulic structure design in preventing the potential scour damage.

Over the last decades, a considerable amount of effort has been devoted to such a subject to estimate the scour depth and thus provide prediction for the extent of necessary protection (Abt et al. 1987; Ade and Rajaratnam 1998; Ali and Lim 1986; Chiew and Lim 1996; Faruque et al. 2006; Lim and Chin 1993; Opie 1967; Rajaratnam and Berry 1977; Rajaratnam and Diebel 1981; Rajaratnam and Humphries 1983; Sarathi et al. 2008). The nature of culvert scour has also been studied intensively (Abida and Townsend 1991; Breusers and Raudkivi 1991; Hoffmans and Verheij 1997). Some of the investigations, such as Bohan (1970) and Ruff et al. (1982), contributed to formulation of engineering guidelines (May et al. 2002; Thompson and Kilgore 2006). Previous laboratory investigations have been conducted under extensive experimental configurations, such as different culvert shapes (circular, triangular, trapezoidal, etc.), different incoming flow conditions (full-flowing or non-full-flowing), different tailwater conditions (unsubmerged or deeply submerged), and different grain sizes and properties (uniform or nonuniform distributions). Consequently, the variety of the test conditions lead to an overwhelming amount of empirical equations available in the published literature (Ade and Rajaratnam 1998; Lim 1995). However, one may find difficulty in choosing a universal formula capable of predicting the scour depth under different flow and sediment conditions.

This study first compiled a database of 588 sets of experimental data, including both published and unpublished ones, and then proposed suitable-length scales to characterize the different incoming flows and sediment bed conditions. The rest of this paper is organized as follows: the "Data Compilation" section describes the data compilation; the " Length Scales of Incoming Flow and Grain Size" section discusses the length scales for incoming flow and sediment properties used in scour depth predictions; the "Data Analysis and Results" section presents the results of the data analysis using the proposed length scales; the "Discussion" section discusses the concerns with the tailwater and scale effects; and the "Conclusion" section concludes the paper.

Data Compilation

Current investigation involves laboratory-scale experiments and compilation of literature data. Altogether, 588 sets of jet and culvert scour data were compiled from experiments conducted in the Hydraulics Modeling Laboratory of Nanyang Technological University (NTU), Singapore, and from the published literature. The study scope is limited to the equilibrium scour of noncohesive sediments under a single horizontal outlet without any offset distance. It is acknowledged that for scour experiments, it is almost impossible to scale the sediment size in laboratory studies. This is because the sediment grains, if scaled based on prototype size, would be in cohesive sediment range, which will result in a different scouring mechanism. For this reason, the compiled data cover a wide range of sediment sizes (0.25–8.7 mm) to minimize the possible sediment size effect. The database can be categorized into two groups in terms of outlet conditions, namely, full-flowing and non-full-flowing types. The published data largely concern circular jet with full-flowing outlet conditions. The air jet data

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(Ade and Rajaratnam 1998; Chiew and Lim 1996; Rajaratnam and Berry 1977) are also included in this analysis for verification purposes.

A summary of the database is listed in Tables 1–3, which tabulates the NTU full-flowing data, the NTU non-full-flowing data, and the data collected from the literature, respectively. It can be seen that the NTU experiments, which contributed more than half of the database, were conducted using a wide range of culvert models and sediment combinations. In addition, the NTU database was collected over a period of more than 10 years and have not been

Table 1. Summary of NTU full-flowing data

Source	Type	Shape	d_o (mm)	b (mm)	Side slope	d_{50} (mm)	σ_g	H (mm)	U_o (m/s)	No. of runs
Tay (1996)	F	Circular	15	—	—	8.7	1.11	18–200	2.7–8.46	26
Seah (1997)	F	Circular	40	—	—	8.7	1.11	20–500	1.35–4.47	26
Lau (1998)	F	Circular	5	—	—	2.35–2.68	1.33–2.16	150	6.08–12.61	28
Chia (2000)	F	Circular	5.92–16.97	—	—	3.97	4.09	200	1.6–12.83	27
Tan (2003)	F	Circular	25.4	—	—	1.6	1.13	250	1.16–2.47	6
Zaihan (2014)	F	Circular	4	—	—	0.77–1.55	1.15–1.2	30–120	1.501–2.15	13
Pee (2016)	F	Circular	8–20	—	—	0.77	1.165	120–300	0.503–2.099	19
Salam (2016)	F	Circular	27.5	—	—	0.5	1.45	13.75–110	0.73–1.258	16
Current data	F	Circular	15–25.4	—	—	0.25–8.7	1.11–3.26	6–200	1.01–5.09	46

Note: F = full-flowing flow conditions; d_o = diameter of circular outlet or water depth of noncircular outlets; b = culvert width; d_{50} = median grain diameter; σ_g = geometric standard deviation of grain size distribution; H = tailwater depth; and U_o = mean velocity of incoming flow.

Table 2. Summary of NTU non-full-flowing data

Source	Type	Shape	d_o (mm)	b (mm)	Side slope	d_{50} (mm)	σ_g	H (mm)	U_o (m/s)	No. of runs
Leow (2006)	NF	Rectangular	—	78	—	1.76–3.8	1.13–3.28	12–17	0.456–0.715	21
Peh (2007)	NF	Rectangular	—	160	—	3.1	3.56	23–38	0.824–1.007	14
Tan (2009)	NF	Trapezoidal	—	120	1.732	4.9	1.27	23–39	0.677–1.013	13
Ma (2010)	NF	Trapezoidal	—	120	1.732	5.2	1.26	32–38	0.815–0.915	4
Theodosius (2012)	NF	Triangular	—	—	1	1.63	1.19	18.8–55.8	0.544–0.778	21
Tan (2013)	NF	Rectangular	—	50	—	1.63	1.19	10–37	0.43–1.116	13
Akhtar (2014)	NF	Rectangular	—	50	—	1.803	2.67	6.5–34.5	0.68–1.003	14
Surya (2014)	NF	Trapezoidal	—	100	2.5	1.622	3.71	20.75–45.85	0.508–0.734	8
Miao (2015)	NF	Trapezoidal	—	100	2.5	1.686	3.83	17.5–81.4	0.32–0.731	11
Das (2015)	NF	Trapezoidal	—	50	1	0.49	1.47	10.25–50.25	0.301–0.694	16
	NF	Rectangular	—	50	—	0.49	1.47	39.6–40.85	0.337–0.455	4
Current data	NF	Rectangular	—	50	—	0.5	1.45	5.9–101.5	0.291–0.723	32

Note: NF = non-full-flowing flow conditions.

Table 3. Summary of complied data from published literautre

Source	Type	Shape	d_o (mm)	b (mm)	Side slope	d_{50} (mm)	σ_g	H (mm)	U_o (m/s)	No. of runs
Lim and Chin (1993)	F	Circular	15	—	—	1.65	1.25–2.5	200	0.79–4.55	17
Chiew and Lim (1996)	AIR	Circular	4	—	—	0.94	1.29	—	36.63–151.65	8
	F	Circular	12.7–25.4	—	—	0.25–1.65	1.25–1.44	200–550	0.53–5.78	33
Opie (1967)	F	Circular	309–914.4	—	—	25.3–204.2	0.99–1.18	122–488	2.18–4.21	12
Rajaratnam and Berry (1977)[a]	AIR	Circular	6.35–23.5	—	—	1.4	1.26–1.5	—	10.058–54.864	26
	F	Circular	25.4	—	—	1.4	1.5	610	1.28–1.81	4
Rajaratnam and Diebel (1981)[a]	F	Circular	12.7–25.4	—	—	1.05	1.5	2.5–43	0.41–2.32	17
Rajaratnam and Humphries (1983)[a]	F	Rectangular	31	50	—	0.11	1.5	31	0.089–0.404	8
Ade and Rajaratnam (1998)	F	Circular	5–25.4	—	—	0.24–7.2	1.33–1.46	14.25–620	2.2–5.5	12
	AIR	Circular	12.5	—	—	1.47	1.12	—	20.3–80	7
Abt et al. (1987)	F	Square	100	—	—	1.86	1.33	—	1.30–3.70	19
		Rectangular	100	160	—					
		Rectangular	100	200	—					
		Arch	100	150	—					
Ali and Lim (1986)	F	Square	50.8	—	—	0.82	1.13	50.8	0.112–0.947	7
Faruque et al. (2006)	F	Square	26.6–76	26.6–76	—	2.46	1.24	53.2–456	0.78–1.31	9
Sarathi et al. (2008)	F	Square	26.6	26.6	—	0.71–2.46	1.14–1.24	26.6–478.8	0.42–2	31

[a]Cases where unrecorded σ_g is assumed as 1.5.

published publicly. "Current data" refers to those collected by the first author during her Ph.D. study. More detailed information of experimental configurations can be found in Tan (2018). The details of these data are given in the complementary file.

Length Scales of Incoming Flow and Grain Size

Dimensional analysis is a common tool in fluid mechanics and hydraulic engineering for scaling fluid systems. The resulting dimensionless number is expected to characterize not merely the relationship among different scales but also the underlying physical meaning, for which suitable velocity and length scales are needed. In the context of culvert scour, typical length scales should be able to represent the properties of the culvert shape, incoming flow conditions, as well as the sediment properties. In practice, the culvert shapes are routinely circular, square, arch, rectangular, etc. (Abt et al. 1987). For a circular culvert, the outlet diameter is usually chosen to represent both the dimension of the culvert shape and the incoming flow. This is straightforward and reasonable as long as the water is flowing fully at the outlet. However, when the outlet flow depth is less than the circular pipe diameter, which is referred to as a "non-full-flowing condition," or when the jet or culvert is not circular, the "diameter" may not be suitable or not even available. For the former case, Ade and Rajaratnam (1998) defined an equivalent pipe diameter, $\sqrt{4A/\pi}$, where A is the area of flow at the outlet. For the latter, Abt et al. (1987) applied the hydraulic radius in the definition of discharge intensity, and Abida and Townsend (1991) simply implemented the culvert height in the definition of the Froude number in their study of rectangular culvert scour. Hitherto, there is no consensus about which length scale should be used for various types of culvert scour. On the other hand, when studying two-dimensional (2D) and three-dimensional (3D) jet scour problems, some researchers suggested that their scouring processes have a similar nature, regardless of the outlet type and flow condition (Ali and Lim 1986; Breusers and Raudkivi 1991; Schiereck 2004). By using the hydraulic radius as a length scale, Ali and Lim (1986) proposed a unified equation for predicting the maximum scour depth induced by deeply submerged 2D and 3D wall jets. Moreover, Breusers and Raudkivi (1991) and Schiereck (2004) compared the plane jet scour, circular jet scour, and culvert scour by plotting the maximum scour depth with the flow intensity, reporting that clustered data for different types of scour exhibit similar trends while only differing in the slope. In light of this, considering that the water jet ensuing from the culvert barrel is essentially a 3D wall jet and the 2D plane jet could be regarded as a limiting case where the culvert base is sufficiently large, one may expect that it is possible to unify different kinds of culvert scour by adopting a suitable length scale. Given that the use of the single-length scale of hydraulic radius to characterize the geometry of the cross-sectional flow has been extensively verified (Gioia and Bombardelli 2002), the following analysis will use the same parameter for describing the outlet flow for both full-flowing and non-flowing in different culvert shapes.

Another important length scale in scour problems is the grain size. For example, the median grain diameter, d_{50}, is often included in the particle densimetric Froude number ($\mathrm{F}_o = U_o/\sqrt{(\rho_s - \rho/\rho)gd_{50}}$, where U_o = mean velocity of incoming flow; ρ and ρ_s are density of water and grain, respectively; and g = gravitational acceleration) in local scour research (Ade and Rajaratnam 1998). To define the particle densimetric Froude number, different grain sizes have been used in the literature. For uniform sediments, the median particle size of d_{50} is commonly used in the scour prediction (Ade and Rajaratnam 1998; Ali and Lim 1986; Lim 1995; Rajaratnam and Berry 1977). For nonuniform sediments, coarser sediment grains may dominate the scouring action. For instance, Hoffmans (1998) used d_{90} to represent sediment property for natural sediment mixtures; and Pagliara et al. (2008) used d_{90} to define F_o and included σ_g ($= \sqrt{d_{84}/d_{16}}$, where d_{16} and d_{84} are the 16th and 84th percentiles of grain size distribution, respectively) as a separate parameter for 2D and 3D plunge pool scour predictions. Moreover, Abida and Townsend (1991) proposed an effective grain size that was calculated from every 10th percentile of grain size distribution. Whenever a grain size other than d_{50}, or σ_g, is used, it is expected that the nonuniformity of sediment is considered to some extent. The inconsistency of grain size chosen for uniform and nonuniform sediments necessitates a suitable representative grain size, which measures both the central tendency and the variability of the grain size distribution. The surface roughness of a channel bed is a function of bed sediment properties, which directly influences channel resistance and thus the vertical profile of flow velocity. It is therefore conjectured that the representative particle size that quantifies the roughness of the scouring sediment bed would be a suitable choice for the purpose of the current study. The previous studies show that a certain percentile of grain size distribution, together with the geometric standard deviation, can be used to describe the nonuniformity of bed sediment. For scouring processes, the inertia of sediment bed could be better represented using a single representative grain size. Such an attempt was made by Cheng (2016), who discussed the equivalent roughness height of a sediment bed by assuming that the grain size follows the lognormal distribution. A similar approach will be used in the following discussion.

Data Analysis and Results

Fig. 1 illustrates the schematic diagram of the culvert scour at the equilibrium state. The maximum scour depth of an equilibrium scour hole downstream of the culvert structure is expected to be affected by several parameters related to incoming flow, flume configuration, and sediment properties. It can be expressed as follows:

$$d_{se} = f(d_o, b, \mu, \rho, U_o, H, B, d_{50}, \sigma_g, \rho_s) \tag{1}$$

where d_{se} = maximum scour depth; d_o = diameter of circular outlet or brink water depth of noncircular outlets; b = base width of noncircular outlets; μ = dynamic viscosity of water; ρ = density of water; U_o = mean velocity of the incoming flow; H = tailwater depth, B = downstream channel width; d_{50} = median grain size; σ_g = geometric standard deviation of the grain size distribution; and ρ_s = density of grain.

As discussed, the hydraulic radius (R_h) can be used to measure the dimension of the incoming flow. In the following analysis, R_h is calculated as the flow area at the brink section divided by its wetted perimeter and used as an integrated length scale to replace d_o and b to incorporate the effects of both culvert shape and full/non-full-flowing outlet conditions. It is also assumed that the downstream channel width (B) is sufficiently wide and its effect on the scour depth can be ignored. Moreover, the viscous effect also can be disregarded by assuming a fully turbulent flow condition. Consequently, by performing dimensional analyses, Eq. (1) can be reformulated as

$$\frac{d_{se}}{R_h} = f\left(\mathrm{F}_o, \sigma_g, \frac{H}{R_h}, \frac{d_{50}}{R_h}\right) \tag{2}$$

In Eq. (2), R_h is used to scale all length parameters; $\mathrm{F}_o\{= [U_o/\sqrt{(\rho_s - \rho/\rho)gd_{50}}]\}$ is the particle densimetric Froude number

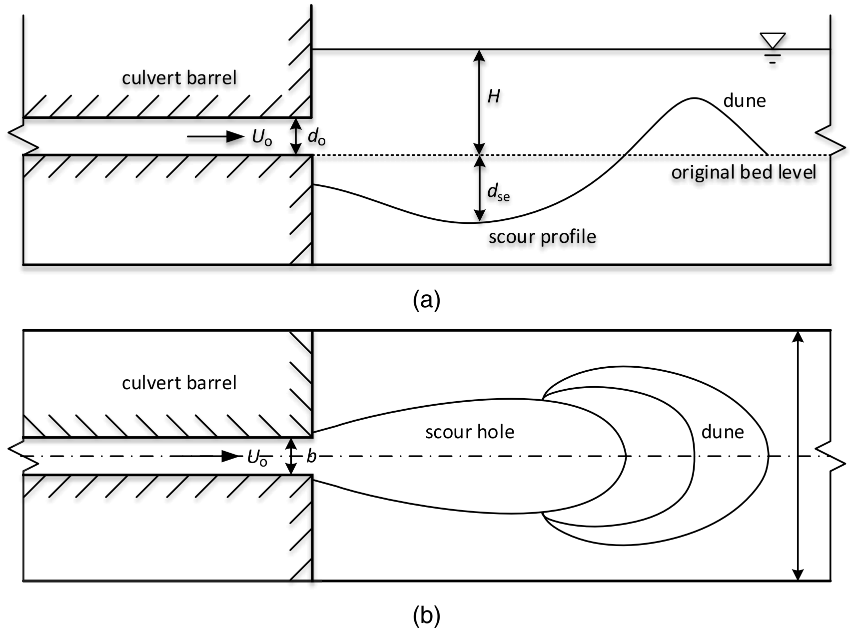

Fig. 1. Definition sketch of a typical culvert scour hole at equilibrium condition.

defined based on d_{50}; and σ_g is used to represent the nonuniformity of sediment.

Previous studies have shown that the geometrical dimension of the equilibrium scour hole caused by 3D wall jets is a function of the particle densimetric Froude number (Ade and Rajaratnam 1998; Chiew and Lim 1996; Lim 1995; Rajaratnam and Berry 1977; Rajaratnam and Diebel 1981). The particle densimetric Froude number is given by a ratio of average flow intensity to the settling velocity of sediment grain, and if d_{50} is included, the settling velocity is only associated with the median grain size. For scouring processes, the scour depth provides a measure of dynamic equilibrium between the bed inertia (or resistance) and the flow intensity. For nonuniform sediment, the bed resistance can be better described using the representative grain diameter rather than the median grain diameter. If the grain size is log-normally distributed, the representative grain diameter can be expressed as

$$d_x = d_{50}\sigma_g^n \tag{3}$$

where n = an exponent. For example, if $n = 1$, $d_x = d_{84}$, and if $n = 2.33$, $d_x = d_{99}$. Generally, the exponent can be calculated using the following formula (Cheng 2016):

$$n = \sqrt{2}\left[\mathrm{erf}^{-1}\left(\frac{x-50}{50}\right)\right] \tag{4}$$

where erf^{-1} = inverse error function. With this consideration, it is proposed that F_o is modified as

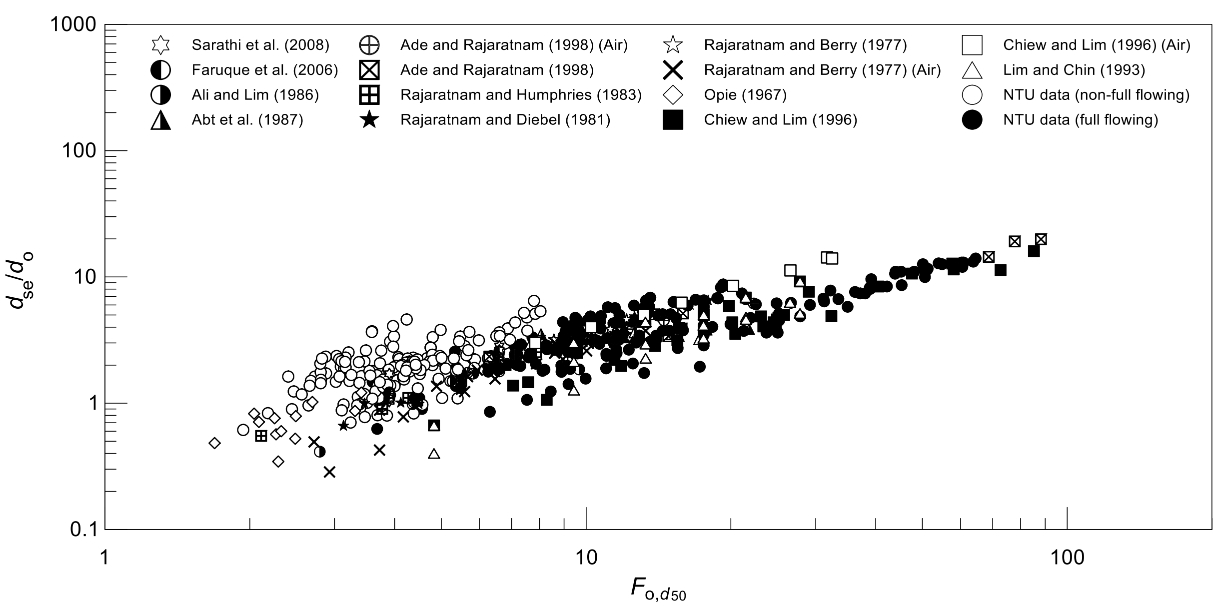

Fig. 2. d_{se}/d_o versus $\mathrm{F}_{o,d_{50}}$ for compiled culvert and jet scour data.

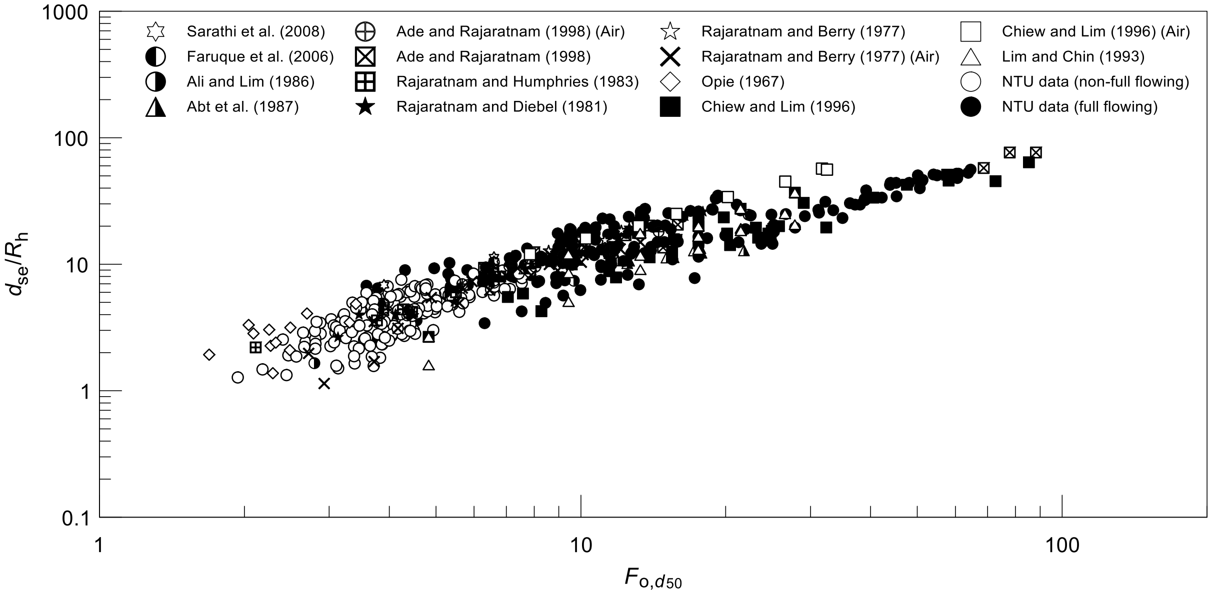

Fig. 3. d_{se}/R_h versus $\mathrm{F}_{o,d_{50}}$ for compiled culvert and jet scour data.

$$\mathrm{F}_{o,d_x} = \frac{U_o}{\sqrt{\frac{\rho_s-\rho}{\rho} g d_x}} \tag{5}$$

In the subsequent data analysis, Eq. (4) is used to calculate various grain size percentiles ranging from d_{55} to d_{99}. Then, their corresponding particle densimetric Froude number F_{o,d_x} is calculated using Eq. (5). Next, regression analysis is performed to find the most suitable representative grain size d_x.

For the purpose of comparison, the maximum scour depth is first normalized by the commonly used d_o and plotted against the d_{50}-based particle densimetric Froude number as shown in Fig. 2. It can be observed that the data points basically show a single cluster with a monotonic increasing relationship trend despite the various outlet conditions, outlet shapes, sediment properties, and tailwater conditions. This observation implies that the particle densimetric Froude number is the most influential parameter and would be effective for predicting the scour depth. One may surmise, therefore, that other parameters such as H/R_h and d_{50}/R_h [shown in Eq. (2)] are of less importance and can be ignored in practice. Next, by replacing d_o with R_h, Fig. 3 clearly shows that the data associated with the non-full-flowing cases (i.e., NTU non-full-flowing) exhibit a better collapsing trend than that shown in Fig. 2. This observation confirms the previous conjecture that R_h is a better length scale for culvert scour problems of various outlet shapes and incoming flow conditions.

Because Fig. 3 is plotted on a log-log scale, it is reasonable to fit the data cluster into the form of $d_{se}/R_h = \alpha \mathrm{F}_o^{\beta}$, where α and β are coefficients. As for the commonly used d_{50}-based particle densimetric Froude number, $\mathrm{F}_{o,d_{50}}$, the regression analysis yields a R^2 value of 0.859. To further explore the suitable representative particle size, a series regression analysis covering a wide range of F_{o,d_x}, where d_x varies from d_{50} to d_{99}, is conducted. The results with R^2 of each F_{o,d_x} are shown in Fig. 4. From the analysis, R^2 improves as d_x increases up to the best R^2 for d_x between d_{80} and d_{85}. Considering both the statistical output and the practicality of the equation, the 84th percentile of grain size, i.e., d_{84}, is determined to be the most suitable representative grain diameter. More explicitly, replacing the $\mathrm{F}_{o,d_{50}}$ in Fig. 3 with the modified $\mathrm{F}_{o,d_{84}}$, Fig. 5 shows that the collapsing trend of the data points is further improved. Consequently, a modified particle densimetric Froude number, $\mathrm{F}_{o,d_{84}}$, using d_{84} as the characteristic grain size, is adopted in the regression analysis and yields

$$\frac{d_{se}}{R_h} = 1.28 \mathrm{F}_{o,d_{84}} \tag{6}$$

From Eq. (6), it is interesting to note that the best fitting result produces a linear relationship between d_{se}/R_h and $\mathrm{F}_{o,d_{84}}$ (with $R^2 = 0.907$ and standard error $= 0.113$). This is consistent with the results of Rajaratnam and Berry (1977), Rajaratnam and Diebel (1981), Lim (1995), Chiew and Lim (1996), and Ade and Rajaratnam (1998), as mentioned. The $\pm 30\%$ error lines are also included in the figure to better assess the accuracy of Eq. (6). For conservative design purposes, $+60\%$ of the estimation of Eq. (6) (blue dash line in Fig. 5) may be referred to as the upper limit of the data cluster.

Furthermore, Eq (6) shows that an estimation of the maximum scour hole depth can be based on $\mathrm{F}_{o,d_{84}}$ alone, by which it also has eliminated the need to include σ_g separately. This analysis shows that R_h and $\mathrm{F}_{o,d_{84}}$ together can unify the data compiled across

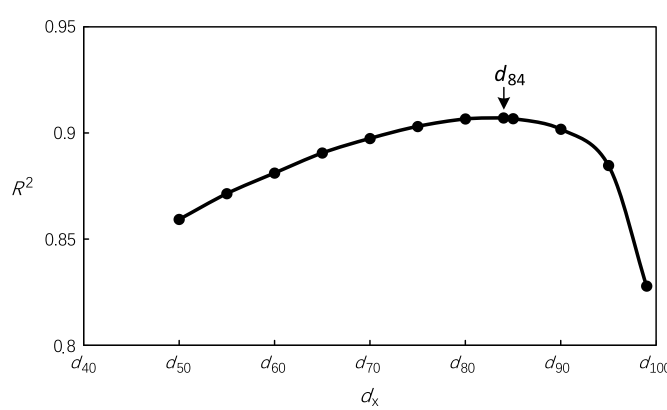

Fig. 4. Regression analysis results using different representative grain sizes.

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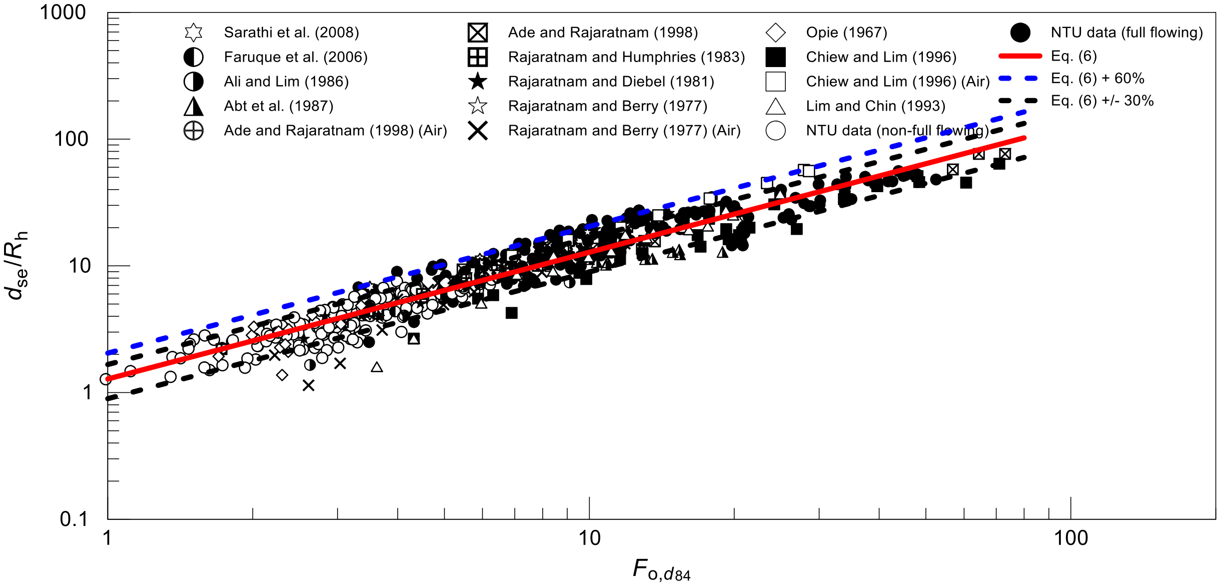

Fig. 5. d_e/R_h versus $F_{o,d_{84}}$ for compiled culvert scour data. The solid line in the figure is Eq. (6); the black dash lines denote ±30% error; and the blue dash line represents the upper limit envelope line.

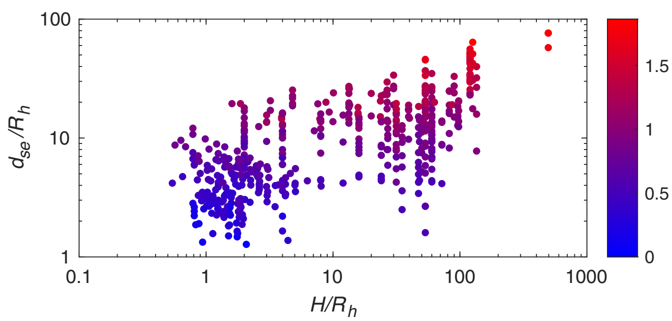

Fig. 6. d_{se}/R_h plotted against H/R_h, where the color bar denotes the value of $\log F_{o,d_{84}}$.

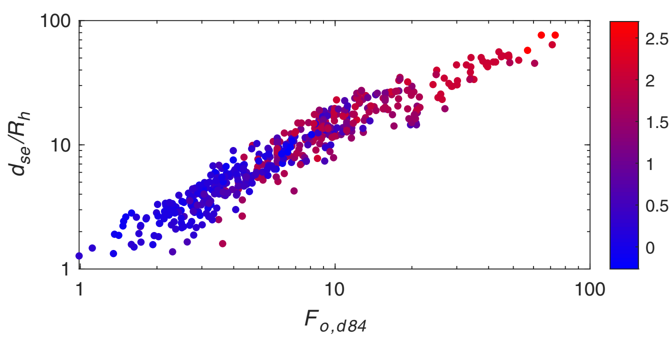

Fig. 7. Plot of d_{se}/R_h versus $F_{o,d_{84}}$, where the color bar denotes the value of $\log(H/R_h)$.

different experimental conditions, which further confirms that other dimensionless numbers commonly used in 3D jet scour studies are of less importance. On a side note, the same representative grain size of d_{84} is often found to be acceptable for characterizing flow resistance in field data analyses (Cheng 2016).

Discussion

The goodness of Eq. (6) in describing the data cluster confirms the conjecture that the particle densimetric Froude number plays the most dominating role in culvert scour estimation. In highlighting this point, the present study is not meant to discount the influences of other effects associated with culvert shape, sediment size, and tailwater depth, etc. As elaborated previously, it is believed that the culvert shape and sediment size effects have been implicitly incorporated in the use of R_h and d_{84}, respectively. To address the tailwater effect, d_{se}/R_h is plotted against H/R_h in Fig. 6, in which the color bar denotes the value of $\log F_{o,d_{84}}$. It can be clearly seen from the figure that d_{se}/R_h exhibits no meaningful variation with H/R_h, suggesting no strong dependence of scour depth on the tailwater depth. Alternatively, by coloring the data point markers with reference to $\log(H/R_h)$, Fig. 7 reveals that for a given $F_{o,d_{84}}$ the data points associated with different H/R_h (by color) appear to be well mixed in the collapsed data cluster. This observation again indicates that for the data used, the tailwater effect is insignificant since organized color layers would be expected if H/R_h is influential.

Moreover, it should be reiterated that for the sake of simplicity, the analysis in the current study is limited to the scope of noncohesive sediments and a single outlet with no offset, which are often adopted in laboratory studies. However, in practice, more complex situations, such as multiple barrel culverts, cohesive soils, and various outlet configurations, are routinely encountered. Given this, extrapolating the present results to different flow and soil conditions could be risky. The application of the proposed relation is subjected to further validation or modification against site-specific prototype information, which requires further efforts in future studies; for example, potential studies performed in the field of sediment transport may be improved by using the knowledge of soil mechanics, particularly for fine-grained sediments.

Conclusion

This paper proposed two length scales, one to represent the dimension of the incoming flow and the other for sediment properties for culvert scour problems. The hydraulic radius is shown to be useful for both full-flowing and non-full-flowing culvert scour with different outlet shapes. By using d_{84} as the representative grain diameter, a modified particle densimetric Froude number $F_{o,d_{84}}$ is proposed to capture the nonuniformity of sediments. This study was performed by assuming that the channel at the outlet is near horizontal and there is no offset distance between the outlet and original sediment bed level. The analysis performed with 588 data sets shows that the maximum scour depth normalized with the hydraulic radius of the incoming flow can be linearly related to the particle densimetric Froude number defined based on d_{84}.

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

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Notation

The following symbols are used in this paper:

A = flow area at the culvert brink section;
B = downstream channel width;
F_o = particle densimetric Froude number;
$F_{o,dx}$ = d_x-based particle densimetric Froude number;
H = tailwater depth;
R_h = hydraulic radius;
U_o = mean velocity of the incoming flow;
b = culvert width;
d_o = diameter of circular outlet or water depth of noncircular outlets;
d_{se} = maximum scour hole depth at equilibrium stage;
d_{50} = median grain diameter;
d_{16} = 16th percentile of grain size distribution;
d_{84} = 84th percentile of grain size distribution;
d_{90} = 90th percentile of grain size distribution;
d_x = xth percentile of the grain size distribution;
g = gravitational acceleration;
μ = dynamic viscosity;
ρ = water density;
ρ_s = grain density; and
σ_g = geometric standard deviation of the grain size distribution.

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